Model reduction of nonlinear systems – use of multi-time scale models and model reduction techniques

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Concerned with large-scale non-linear circuits.

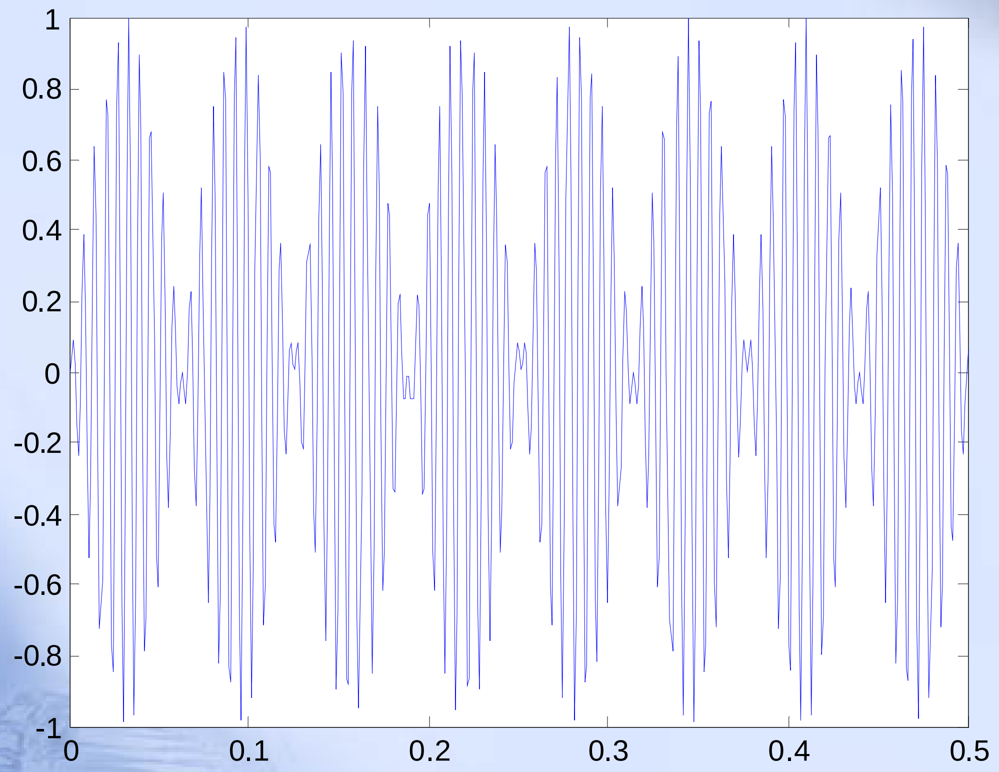
Their behaviour is described by:

$$\dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t)) + \boldsymbol{b}(t)$$

$\boldsymbol{x}(t)$ are the states

$\boldsymbol{b}(t)$ are the inputs to the circuit

Concerned with envelope modulated signals



Such systems are suitable for MULTI-TIME SCALE analysis

We set $x(t) = \hat{x}(t_1, \cdots t_p)$

p is the number of different time scales

We rewrite the governing equation in multi-time scale format

$$\frac{\partial \hat{x}}{\partial t_1} + \frac{\partial \hat{x}}{\partial t_2} + \cdots \frac{\partial \hat{x}}{\partial t_p} = f(\hat{x}(t_1, t_2 \cdots t_p)) + \hat{b}(t_1, t_2 \cdots t_p)$$

We can solve the multi-time scale partial differential equation using:

- Complete time-domain methods
- Mixed time-domain frequency-domain methods
- Mixed time-domain wavelet methods

We will consider wavelet approach

Why? –

- Suitable for highly nonlinear circuits
- Enables us to move from one level of accuracy to another in an incremental manner

To simplify matters, consider two time scales.

$$\frac{\partial \hat{x}}{\partial t_1} + \frac{\partial \hat{x}}{\partial t_2} = f(\hat{x}(t_1, t_2)) + \hat{b}(t_1, t_2)$$

$x(t)$ is evaluated along the diagonal of the t_1, t_2 plane

$$x(t) = \hat{x}(t, t)$$

x(t) is represented by:

- a series of wavelets in one time scale
- coefficients which vary in the other time scale

$$\hat{x}_J(t_1,t_2) = \sum_{k=1}^{N} \bar{x}_k(t_1)\Psi_k(t_2)$$

If written for all collocation points in t_2

$$\hat{x}_{JN}(t_1) = E\bar{x}(t_1)$$

E is a constant square matrix.
Its columns comprise the values of the *N* wavelet functions, $\Psi_k(t_2)$,
at the *N* collocation points

The governing equation now becomes:

$$E\frac{d\overline{x}}{dt_1} = -D\overline{x} + f_N(\overline{x}) + b_N$$

This is an ODE – but it concerns ONE time scale –

-so can be solved numerically in an efficient manner

At this point we employ MODEL REDUCTION techniques

We expand $\bar{x}(t_1)$ in a Taylor's series t_1^0 is the initial time

$$\bar{x}(t_1) = \sum_{i=0}^{\infty} a_i (t_1 - t_1^0)^i$$

The coefficients a_i may be computed recursively

A Krylov space is formed from a_i

$$\boxed{K = [a_0 \quad a_1 \quad \cdots \quad a_q]}$$

q is the order of the reduced system

We perform a QR decomposition of the Krylov space

$$\boxed{K = QR}$$

$$Q^T Q = I_q$$

Then Q is employed to perform a congruent transformation:

$$\boxed{\bar{x} = Q\hat{x}}$$

$$Q^T E Q \frac{d\hat{\hat{x}}}{dt_1} = -Q^T D Q\hat{\hat{x}} + Q^T f_N(Q\hat{\hat{x}}) + Q^T b_N$$

or

$$\boxed{\hat{E} \frac{d\hat{\hat{x}}}{dt_1} = -\hat{D}\hat{\hat{x}} + Q^T f_N(Q\hat{\hat{x}}) + \hat{b}_N}$$

where

$$\hat{E} = Q^T E Q \qquad \hat{D} = Q^T D Q \qquad \hat{b}_N = Q^T b_N$$

Reduced system is of size $q << N$ – where N is the original size

For oscillators or systems subject to Frequency Modulated signals we use the Warped Multi-time scale model

$$\omega(\tau_2)\frac{\partial \hat{x}}{\partial t_1} + \frac{\partial \hat{x}}{\partial t_2} = f(\hat{x}(t_1,t_2)) + b(t_2)$$

It bends the path along which $x(t)$ is evaluated to account for frequency variation

$$x(t) = \hat{x}(\varphi(t),t)$$

$$\varphi(t) = \int_0^t \omega(t)dt$$

Again we apply wavelet analysis

$$\hat{x}_J(t_1,t_2) = \sum_{k=1}^{N} \bar{x}_k(t_2)\,\Psi_k(t_1)$$

Warped Equation now becomes:

$$\omega(\tau_2)D\overline{x} + E\frac{\partial \overline{x}}{\partial t_2} = f(E\overline{x}) + b_N(t_2)$$

We apply the same reduction procedure

$$\overline{x}(t_2) = \sum_{i=0}^{\infty} a_i (t_2 - t_2^0)^i$$

$$K = [a_0 \quad a_1 \quad \cdots \quad a_q] = QR$$

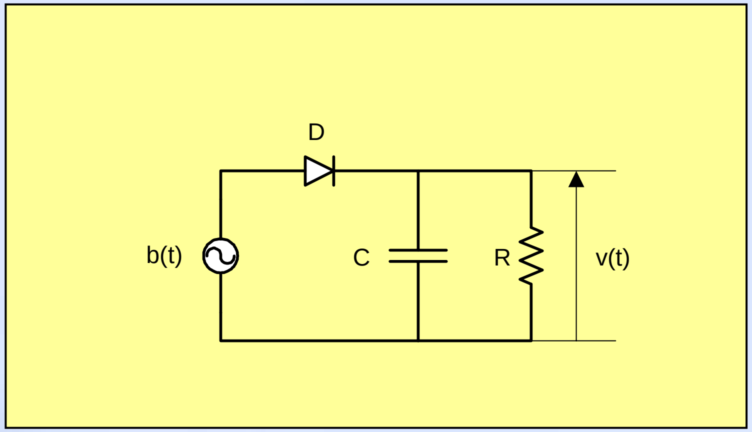
Reduced equation is of size $q<<N$

$$\omega(\tau_2)\hat{D}\hat{\hat{x}} + \hat{E}\frac{\partial\hat{\hat{x}}}{\partial t_2} = Q^T f(Q\hat{\hat{x}}) + \hat{b}_N(t_2)$$

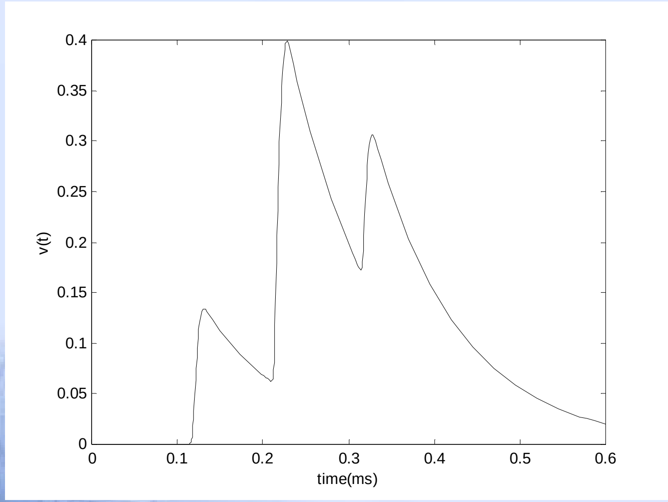
Example:

Input signal: $$b(t) = sin\left(\frac{2\pi}{T_1}t\right)sin\left(\frac{2\pi}{T_2}t\right)$$

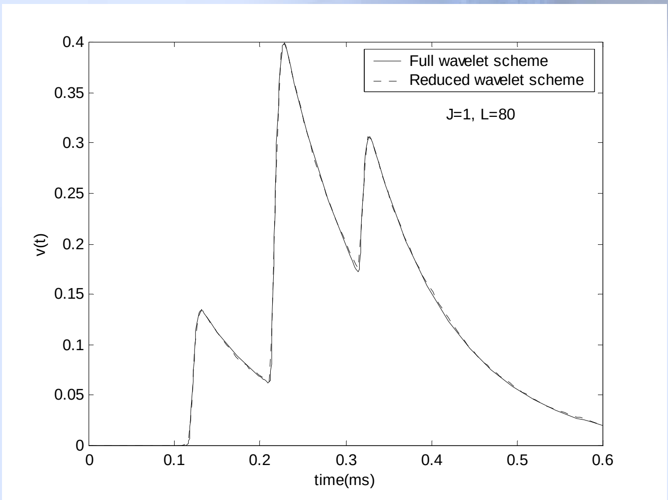
Nonlinear circuit



Output with full model



Output from reduced wavelet model



CONCLUSIONS

- Multi-timescale analysis suitable for nonlinear circuits with modulated signals

- Multi-timescale model can be reduced using Krylov space techniques

MORE RESEARCH WORK REQUIRED
FOR NONLINEAR MODEL REDUCTION

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