Low-dimensional manifolds in reaction-diffusion equations

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The approach to equilibrium for systems of reaction-diffusion equations on bounded domains is studied geometrically. It is shown that equilibrium is approached via low-dimensional manifolds in the infinite-dimensional function space for these dissipative, parabolic systems. The fundamental aspects of this process are mapped out in some detail for single species cases and for two-species cases where there is an exact solution. It is shown how the manifolds reduce the dimensionality of the system from infinite dimensions to only a few dimensions. For larger systems calculations are undertaken to study the approach to equilibrium. It is demonstrated that a number of systems approach equilibrium along attractive low-dimensional manifolds over significant ranges of parameter space. Numerical methods for generating the manifolds are adapted from methods that were developed for systems of ordinary differential equations. The truncation of the infinite spectrum of the partial differential equations makes it necessary to devise a new version of one of these methods, the well-known algorithm of Maas and Pope.