Patch dynamics: macroscopic simulation of multiscale systems

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For an important class of multiscale problems, a separation of scales exists between the available (microscopic) model and the (macroscopic) level at which one would like to observe and analyze the system. For time-dependent multiscale problems of this type, Kevrekidis et al. developed a so-called "equation-free" framework, based on the idea of a so-called coarse-grained time-stepper. The patch dynamics scheme is a coarse-grained time-stepper which approximates the time evolution of a set of spatially distributed macroscopic variables for which the governing partial differential equation (PDE) is not (or only approximately) available; the scheme only performs appropriately initialized simulations using the available microscopic model in small portions of the space-time domain (the patches).

We present some theoretical and numerical convergence results for a class of parabolic homogenization problems. In particular, we show that the scheme approximates a finite difference scheme for the unavailable macroscopic equation. We also formulate and analyze a finite volume variant, and discuss some issues that arise when trying to combine patch dynamics with adaptive mesh refinement (AMR). We conclude with a brief discussion of the possibilities and limitations of the current schemes, and provide some ideas for future research.