Invariant manifolds stability and stabilisation in lattice Boltzmann methods

R.A. Brownlee, A.N. Gorban, J. Levesley

Department of Mathematics, University of Leicester, UK

We revisit the classical stability versus accuracy dilemma for the lattice Boltzmann methods (LBM). Our goal is a stable method of second-order accuracy for fluid dynamics based on the lattice Bhatnagar-Gross-Krook method (LBGK).

The LBGK scheme can be recognized as a discrete dynamical system generated by free-flight and entropic involution. In this framework the stability and accuracy analysis are more natural. We find the necessary and sufficient conditions for second-order accurate fluid dynamics modeling. In particular, it is proven that in order to guarantee second-order accuracy the distribution should belong to a distinguished surface -- the invariant film (up to second-order in the time step). This surface is the trajectory of the (quasi)equilibrium distribution surface under free-flight.

The main instability mechanisms are identified. The simplest recipes for stabilization add no artificial dissipation (up to second-order) and provide second-order accuracy of the method. We construct a system of nonequilibrium entropy limiters for the lattice Boltzmann methods (LBM). These limiters erase spurious oscillations without blurring of shocks, and do not affect smooth solutions. In general, they do the same work for LBM as flux limiters do for finite differences, finite volumes and finite elements methods, but for LBM the main idea behind the construction of nonequilibrium entropy limiter schemes is to transform a field of a scalar quantity - nonequilibrium entropy. There are two families of limiters: (i) based on restriction of nonequilibrium entropy (entropy ``trimming") and (ii) based on filtering of nonequilibrium entropy (entropy filtering). The physical properties of LBM provide some additional benefits: the control of entropy production and accurate estimate of introduced artificial dissipation are possible. The constructed limiters are tested on classical numerical CFD benchmarks.