From hyperbolic regularization to exact hydrodynamics via simple kinetic models

Colangeli M, Karlin I.V., Kröger M.
ETH Zurich

Abstract

The derivation of hydrodynamics from a microscopic description is the classical problem of physical kinetics. The Chapman-Enskog method derives the solution from the Boltzmann Equation in the form of a series in powers of Knudsen number, given by the ratio between the particle mean free path and the length scale variations of hydrodynamics fields. However, as first demonstrated by Bobylev for Maxwell molecules, even in the simplest case (one-dimensional linear deviations from global equilibrium), the Burnett and the Super-Burnett hydrodynamics violate the basic physics behind the Boltzmann Equation. Namely, the acoustic contributions at sufficiently short wave lengths increase with time instead decaying. Inspired by a recent hyperbolic regularization of Burnett hydrodynamic equations, we introduce a method to derive stable equations of linear hydrodynamics to any desired accuracy in Knudsen number. We first proceed with derivation from a simple kinetic model – a thirteen Moments Grad System – recovering and generalizing [1] the previous Bobylev's result, including the proof of an H-theorem [2]. Further we consistently apply the same technique to derive hydrodynamics from linearized Boltzmann Equation. We demonstrate that stability of macroscopic time evolution equations arises as interplay between two basic features: dissipativity and hyperbolicity.

[1] M. Colangeli, I.V. Karlin, M. Kröger, From hyperbolic regularization to exact hydrodynamics for linearized Grad's equations, Phys. Rev. E 75 (2007) 051204.
[2] M. Colangeli, I.V. Karlin, M. Kröger, Hyperbolicity of exact hydrodynamics for three-dimensional linearized Grad's equations, Phys. Rev. E 76 (2007) in press.