TECHNICAL NOTE

On "Solid Liquid" Limit of Hydrodynamic Equations

In Ref. [1-4] we have obtained new hydrodynamic equations beyond a traditional framework of Knudsen number expansions. Approaches of paper [1] and of papers [3-4] are markedly different. In Ref. [1], a perturbation theory similar to KAM-theory was developed for the Boltzmann equation, and it was based on Newton iterative procedures and parametrix expansions. No asymptotic expansions in powers of Knudsen number were used in [1]. On the other hand, in Ref. [2-4], a partial summation of all terms of Chapman-Enskog expansion for Grad equations was employed. However, results of both approaches have much in common. In particular, for a nonlinear case, a nontrivial threshold behavior of viscous stress tensor at high compression of a flux was detected (see a discussion in [1]). Namely, there exists such negative value of flux divergency, \( \text{div} \mathbf{u}^* \), at which the stress tensor diverges as a function of \( \text{div} \mathbf{u} \). For \( \text{div} \mathbf{u} < \text{div} \mathbf{u}^* \), an effective viscosity is positive, while for \( \text{div} \mathbf{u} > \text{div} \mathbf{u}^* \) it is negative (a non-physical region). In this short communication we show that, thanks to the divergency indicated, a transition from the physical region into that nonphysical is indeed impossible.

An expression for the nonlinear stress tensor [1-4] reads (we consider the one-dimensional case for simplicity):

\[
\sigma \sim -a \frac{\partial_x u}{1 + b \partial_x u}
\]

(1)

Here \( a \) and \( b \) are positive numbers, while \( \partial_x u \) gives \( \text{div} \mathbf{u} \) in dimensions higher than one. Considering \( \sigma \) as a function in the variable \( \xi = \partial_x u \), we see that \( \sigma \to +\infty \) as \( \xi - \xi^* \to +0 \), where \( \xi^* = -\frac{1}{b} \) is the threshold value of \( \xi \). Denote as \( \Delta(x) = \xi(x) - \xi^* \) a deviation from the threshold value \( \xi^* \) and assume that, for some \( x' \), the deviation of \( \xi(x') \) from \( \xi^* \) is small, i.e. \( |\Delta(x')| \to 0 \). Preserving only the leading terms with respect to \( |\Delta| \) in hydrodynamic equations [1-4], we obtain:

\[
\frac{d\Delta(x',t)}{dt} \sim \frac{2a}{b^2} \left( \partial_x \Delta(x,t) \bigg|_{x = x'} \right)^2 \Delta^{-3}(x',t)
\]

(2)
Note that a general expression of the stress tensor obtained in [1] accounts not only the nonlinearity in $\partial_x u$ but also a nonlocality, and in this respect it differs from expression (1), but in the limit considered the corresponding equation for $\Delta$ has again the form of eq.(2).

As it follows from (2), if the initial value $\Delta(x',0)$ is positive (negative), then for $t > 0$ values $\Delta(x',t)$ are also positive (negative). The closer is the initial value of $\xi$ to the threshold value $\xi^*$, the stronger is the repulsion from this threshold. This means that the variable $\xi$ does not leave the physical region $\xi > \xi^*$, and also that transitions from the non-physical region into the physical region do not occur. In other words, the effective viscosity $\mu = 1 - \frac{\alpha}{b \partial_x u}$ becomes infinite as $\xi \to \xi^*$, and thus the physical and the non-physical regions are separated with an "infinitely high barrier". This self-confinement in the physical region due to hydrodynamic equations [1-4] is in a contrast to the Burnett hydrodynamic equations. The latter suffer a transition into the negative viscosity region in a regular point, and thus a solution might loose a sense. On contrary, for hydrodynamic equations [1-4], if a solution starts in a physical region, then it remains there in the following evolution.

A physical interpretation of the "infinitely viscid threshold" is as follows. A rapid compression of a flux amounts to a strong deceleration of particles (particles loose velocity comparable to heat velocity on a distance comparable to the main free path). Such strong deceleration is described in the frames of hydrodynamic equations [1-4] by a divergency of viscosity (a fluid becomes "solid"). This effect of "solid liquid" was first detected in [4] via a regularization of Burnett approximation. A more consequent approach of Ref.[1] gives qualitatively similar results, and its applicability enables one to consider such regimes.

A.N.Gorban
Computing Center,
Krasnoyarsk 660036 Russia
Ulm University,
Ulm 89069 Germany

I.V.Karlin

April 21, 1995

References