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TECHNICAL NOTE

On "Solid Liquid" Limit of Hydrodynamic Equations

In Ref.[1-4] we have obtained new hydrodynamic equations beyond a traditional framework of Knudsen number expansions. Approaches of paper [1] and of papers [3-4] are markedly different. In Ref.[1], a perturbation theory similar to KAM-theory was developed for the Boltzmann equation, and it was based on Newton iterative procedures and parametrix expansions. No asymptotic expansions in powers of Knudsen number were used in [1]. On the other hand, in Ref.[2-4], a partial summation of all terms of Chapman-Enskog expansion for Grad equations was employed. However, results of both approaches have much in common. In particular, for a nonlinear case, a nontrivial threshold behavior of viscid stress tensor at high compression of a flux was detected (see a discussion in [1]). Namely, there exists such negative value of flow divergency, $\mathrm{div}\mathbf{u}^*$ at which the stress tensor diverges as a function of $\mathrm{div}\mathbf{u}$. For $\mathrm{div}\mathbf{u} < \mathrm{div}\mathbf{u}^*$, an effective viscosity is positive, while for $\mathrm{div}\mathbf{u} > \mathrm{div}\mathbf{u}^*$ it is negative (a non-physical region). In this short communication we show that, thanks to the divergency indicated, a transition from the physical region into that nonphysical is indeed impossible.

An expression for the nonlinear stress tensor [1-4] reads (we consider the one-dimensional case for simplisity):

$$\sigma \sim -a\frac{\partial_x u}{(1 + b\partial_x u)} \tag{1}$$

Here a and b are positive numbers, while $\partial_x u$ gives $\mathrm{div}\mathbf{u}$ in dimensions higher than one. Considering σ as a function in the variable $\xi = \partial_x u$, we see that $\sigma \to +\infty$ as $\xi - \xi^* \to +0$, where $\xi^* = -\frac{1}{b}$ is the threshold value of ξ. Denote as $\Delta(x) = \xi(x) - \xi^*$ a deviation from the threshold value ξ^* and assyme that, for some x', the deviation of $\xi(x')$ from ξ^* is small, i.e. $|\Delta(x')| \to 0$. Preserving only the leading terms with respect to $|\Delta|$ in hydrodynamic equations [1-4], we obtain:

$$\frac{d\Delta(x',t)}{dt} \sim \frac{2a}{b^2}\left(\partial_x \Delta(x,t)|_{x\,=\,x'}\right)^2 \Delta^{-3}(x',t) \tag{2}$$

Note that a general expression of the stress tensor obtained in [1] accounts not only the nonlinearity in $\partial_x u$ but also a nonlocality, and in this respect it differs from expression (1), but in the limit considered the corresponding equation for Δ has again the form of eq.(2).

As it follows from (2), if the initial value $\Delta(x',0)$ is positive (negative), then for $t > 0$ values $\Delta(x',t)$ are also positive (negative). The closer is the initial value of ξ to the threshold value ξ^*, the stronger is the repulsion from this threshold. This means that the variable ξ does not leave the physical region $\xi > \xi^*$, and also that transitions from the non-physical region into the physical region do not occur. In other words, the effective viscosity $\mu = \frac{a}{1 + b\partial_x u}$ becomes infinite as $\xi \to \xi^*$, and thus the physical and the non-physical regions are separated with an "infinitely high barrier". This self-confinement in the physical region due to hydrodynamic equayions [1-4] is in a contrast to the Burnett hydrodynamic equations. The latter suffer a transition into the negative viscosity region in a regular point, and thus a solution might loose a sense. On contrary, for hydrodynamic equations [1-4], if a solution starts in a physical region, then it remains there in the following evolution.

A physical interpretation of the "infinitely viscid threshold" is as follows. A rapid compression of a flux amounts to a strong deceleration of particles (particles loose velocity comparable to heat velocity on a distance comparable to the main free path). Such strong deceleration is described in the frames of hydrodynamic equations [1-4] by a divergency of viscosity (a fluid becomes "solid"). This effect of "solid liquid" was first detected in [4] via a regularization of Burnett approximation. A more consequent approach of Ref.[1] gives qualitarively similar results, and its applicability enables one to consider such regimes.

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