

Invariant Manifold-Based Model Reduction for a Class of Nonlinear Discrete-Time Dynamical Systems Using Functional Equations

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ABSTRACT

The present research work proposes a new approach to the model-reduction problem for nonlinear dynamical systems in the discrete-time domain based on the notion of invariant manifold. In particular, we consider nonlinear discrete-time “input-driven” real analytic dynamical systems of a structure reminiscent of skew-symmetric systems. It is implicitly assumed, that the discrete-time nonlinear dynamic models considered can be obtained: (i) either through the employment of efficient and accurate discretization methods for the original continuous-time system/process which is mathematically described by a system of nonlinear ordinary (ODEs) or partial differential equations (PDEs), or (ii) through direct identification methods for the underlying dynamics based on historical data. Furthermore, nonlinear dynamical systems are considered whose dynamics can be viewed as “driven”: (i) either by an external time-varying “forcing” or input/disturbance term, or (ii) by a set of time-varying system/process parameters, or (iii) by the autonomous dynamics of an “upstream” system/process. The formulation of the problem is conveniently realized through a system of nonlinear first-order functional equations (NFEs) and a rather general set of necessary and sufficient conditions for solvability is derived. In particular, within the class of analytic solutions the aforementioned set of conditions guarantees the existence and uniqueness of a locally analytic solution. The solution of the aforementioned system of NFEs is proven to be a locally analytic invariant manifold, and the local analyticity property of the invariant manifold enables the development of a series solution method, which can be easily implemented with the aid of a symbolic software package such as MAPLE. Under a certain set of conditions, it is shown that the invariant manifold computed attracts all system trajectories, and therefore, model reduction (and the asymptotic long-term dynamic behavior) is realized through the restriction of the discrete-time system dynamics on the invariant manifold. Finally, in order to illustrate the proposed approach, a representative biological reactor example is considered.