Model reduction in mathematical neuroscience

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I will discuss two common approaches to model reduction for large scale spiking neural networks, as well as their limitations. The first gives rise to networks of interacting phase-oscillators of Kuramoto type. Here the limitation is the assumption of a strongly attracting limit cycle oscillation and that interactions are weak. This approach builds heavily on the use of the infinitesimal phase response curve (iPRC), and we identify a number of scenarios in which this standard approach breaks down. In particular shear-induced chaos, i.e., chaotic behaviour that results from the amplification of small perturbations by underlying shear, is missed entirely by the iPRC, and highlights the need to develop phase-amplitude models. The second approach is a naive spatio-temporal coarse graining that gives rise to continuum models for whole brain activity, often referred to as neural field models. These attempt to track the average membrane potential in a population utilising a phenomenological nonlinear firing rate function. This mean-field style approach cannot account for the evolution of spike-train correlations and can give misleading predictions when comparing to spiking models with fast synaptic interactions. Finally I will discuss the Lighthouse spiking neural network model of Hermann Haken. This particular model may allow a bridge to be built between spike and rate descriptions. Indeed in the limit of slow synaptic interactions it reduces to the oft-studied Amari neural field model. Importantly the Lighthouse model is sufficiently simple that it may also be analysed directly at the network level, even for fast synaptic responses. Hence, a comprehensive study of a network of synaptically coupled Lighthouse neurons may pave the way for the development of a specific exactly soluble neurodynamics.