Model Reduction of Flow Systems and Networks by the Maximum Entropy Method

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The maximum entropy (MaxEnt) principle of Jaynes (1957, 2003) provides a powerful technique for model reduction in complex dynamical systems. Although imbued with several philosophical interpretations, the most useful is undoubtedly the original viewpoint of Boltzmann (1877), that we seek the most probable state of the system. For a dynamical system subject to fixed constraints, this will coincide with its stationary state. This viewpoint unites the analysis of (a) physical and chemical thermodynamic systems with fixed contents, in which we seek the equilibrium state; and (b) non-equilibrium flow and chemical reaction systems with fixed forcings, for which we seek the steady state. The MaxEnt method therefore enables a tremendous degree of model reduction; e.g. of the order of 23 orders of magnitude in thermodynamic systems.

Recently, the author presented a new formulation of non-equilibrium thermodynamics for the analysis of infinitesimal flow systems, based on a direct application of MaxEnt (Niven, 2009, 2010). The analysis invokes an entropy over the set of instantaneous flow and reaction states, giving a potential function (analogous to the Planck potential) which is minimised at steady-state flow. The analysis is analogous in every way to the MaxEnt formulation of equilibrium thermodynamics (e.g. Callen, 1985). This framework and its implications are first presented in detail. The framework is then extended to the MaxEnt analysis of a flow network (Niven et al., 2013; Waldrip et al., 2013), a representation which crosses many disciplines, including electrical circuit, communications, water distribution, vehicular transport, chemical reaction, ecological and human social systems. The method is illustrated by the analysis of under-constrained pipe flow and traffic flow networks, as specified by sets of flow rates and potentials, some of which are fixed. The MaxEnt method is sufficiently general that it can be extended to handle other uncertainties in the system specification, even in the network structure itself.

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