Reducing a family of attractors: parameter dependence in the reduced model

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Attractors are important features of dynamical systems that are key to understanding long-term behaviour. These objects are often low-dimensional compact shapes, such as limit cycles and tori, living in state spaces that may be of high (or even infinite) dimension. They are therefore natural targets for methods of dimension reduction for dynamical systems. However, it is common for models of physical systems to depend on parameters that can take on different values in different instances of the system. This results in a family of attractors that may feature variation in scale and shape, and bifurcations that produce more significant variation. In order to account for the full range of behaviour possessed by such models, the challenge is to obtain a reduced model that correctly responds to the parameters, producing the family of attractors in a low-dimensional system.

Rather than attempting to manipulate the differential equations directly, we take a geometric approach by regarding the system to be a family of vector fields on a state space manifold. We make use of secant-based projection to reduce the dimension, which smoothly embeds the attractors into the reduced space. By reproducing the relevant parts of the vector field across a region of parameter space, we can produce a reduced model that exhibits the same attractors as the original model, including some types of bifurcation. As a result of this approach, the method is compatible with non-Euclidean state spaces, such as those containing angular variables, and nonautonomous systems with periodic time dependence.

In this talk we give an overview of this approach, describe some of the interesting challenges, and show some examples of the method in action.