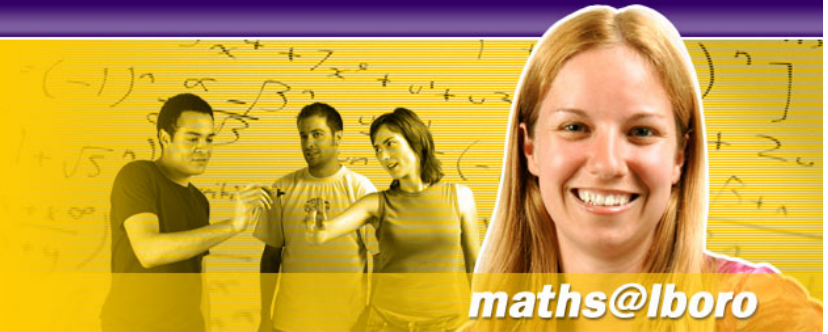


Department  
of Mathematics



# Breathing with delayed feedback

Natalia Janson

with Christine Parkes and Andrey Pototsky

[www.lboro.ac.uk/departments/ma](http://www.lboro.ac.uk/departments/ma)

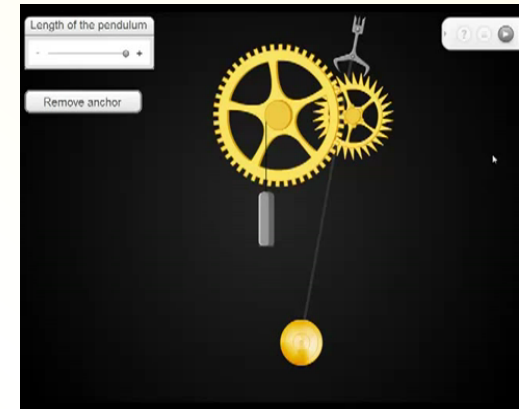


# Processes in cardiovascular system

Heart beats, breathing

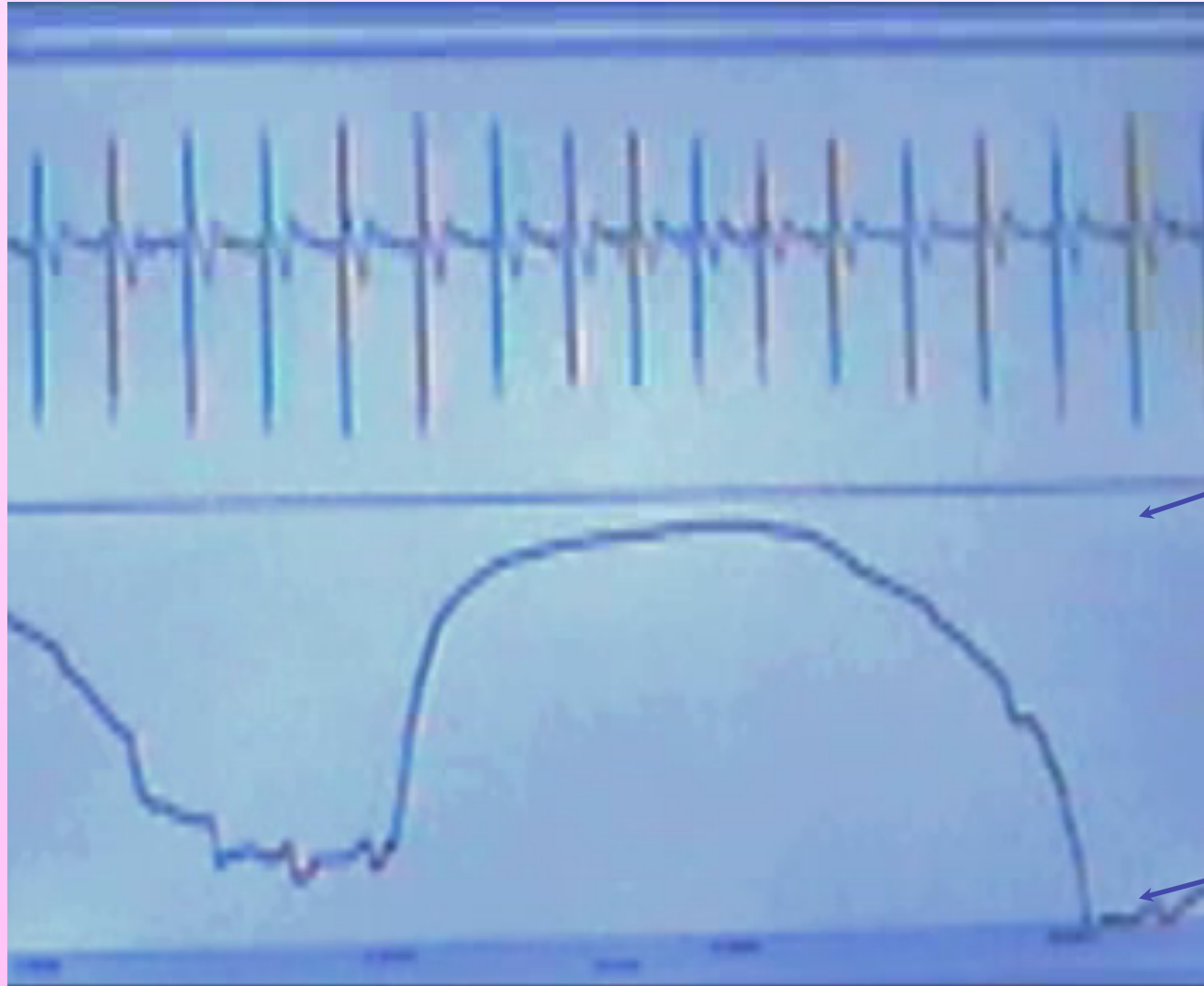
1. Never stop (while the subject lives)
2. Rhythmic, but non-periodic
3. Occurs in a non-linear dissipative system
  - living system

Assume



These processes are self-oscillatory.  
Mathematical models: limit cycles,  
continuously and randomly perturbed

# Inter-beat intervals vary, also due to breathing



electrocardiogram

Breathe-in  
(faster  
heart-beats)

Breathe-out  
(slower  
heart-beats)

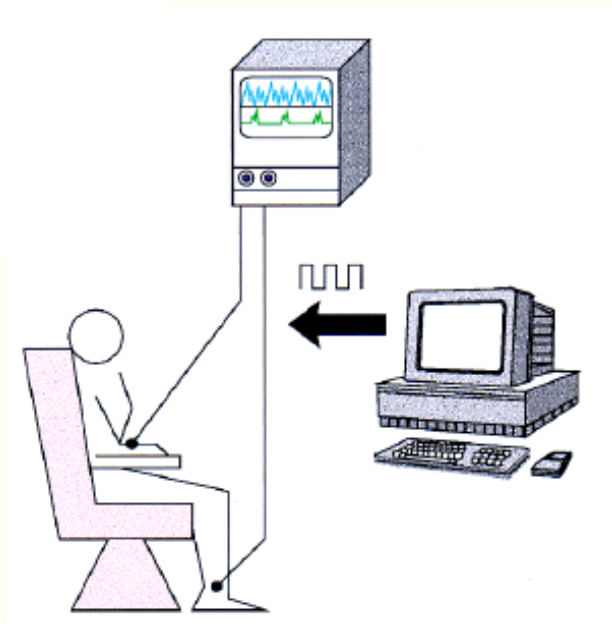


**Inter-beat intervals  
vary (sound)**



# Previous studies with heart beats: 1

Weak visual and auditory rhythmic stimuli can change heart beats in healthy volunteers.



[V.S. Anishchenko, A.G. Balanov, N.B. Janson, N.B. Igosheva, G.V. Bordyugov, Int. Journal of Bifurcation and Chaos 4, 2339 (2000) ]

[V.S. Anishchenko, A.G. Balanov, N.B. Janson, N.B. Igosheva, G.V. Bordyugov, Discrete Dynamics in Nature and Society 4, 201 (2000) ]

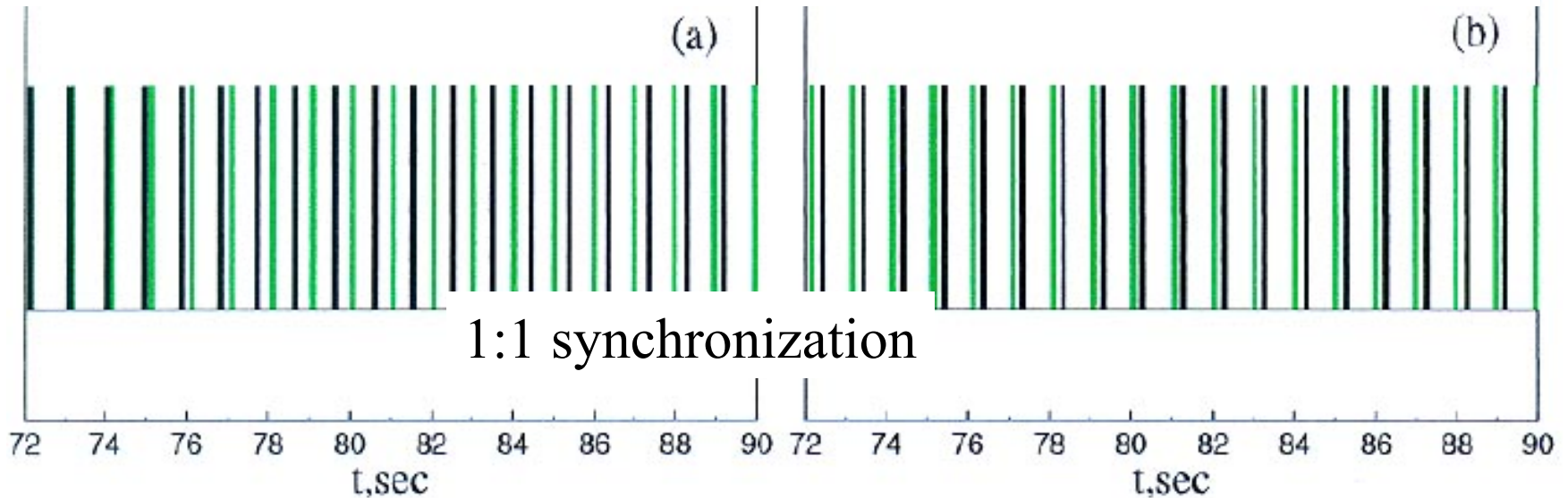


# Forced synchronisation of heart beats

Black – unforced, Green - forcing

Black – forced, Green - forcing

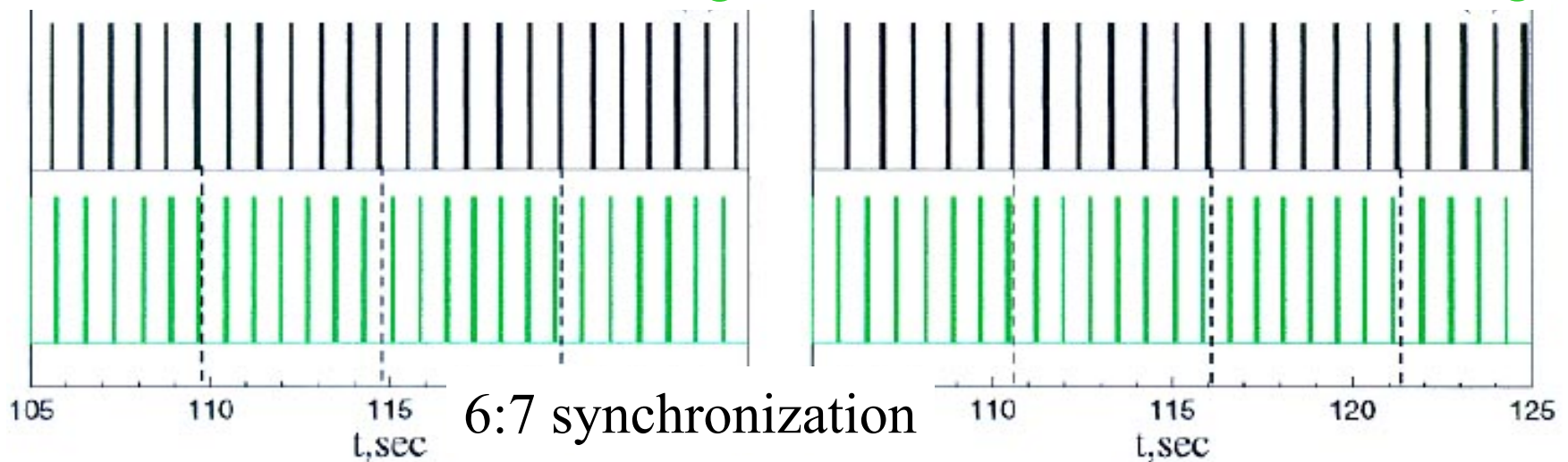
Periodic beeping  
Zero detuning



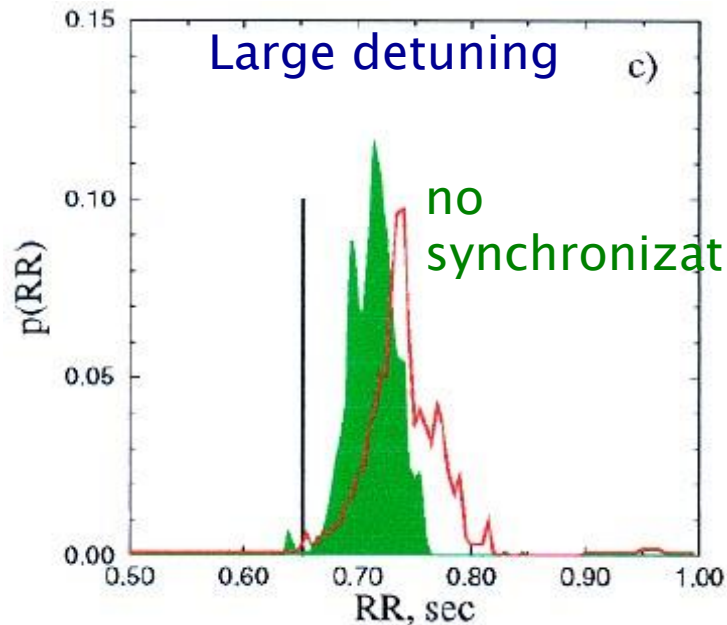
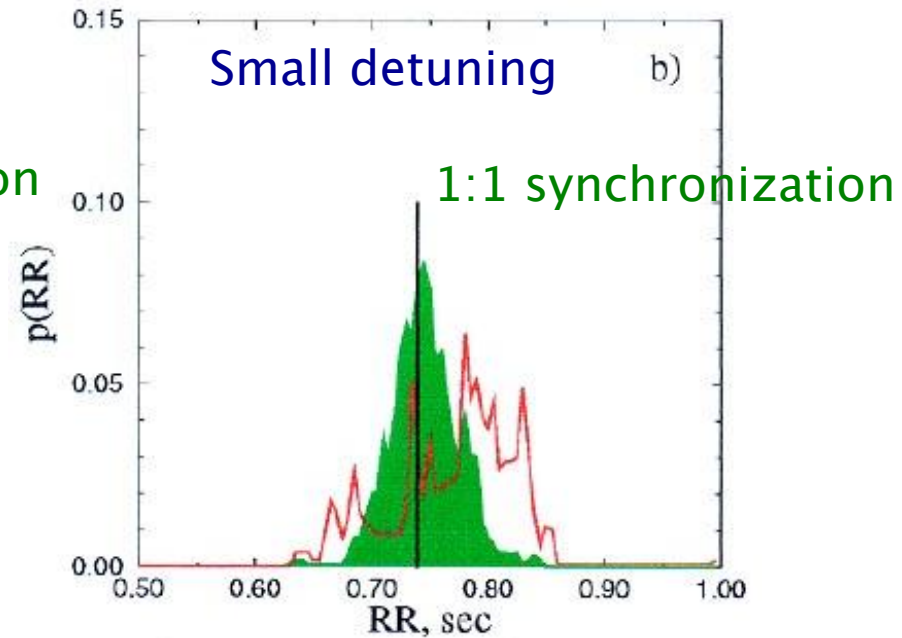
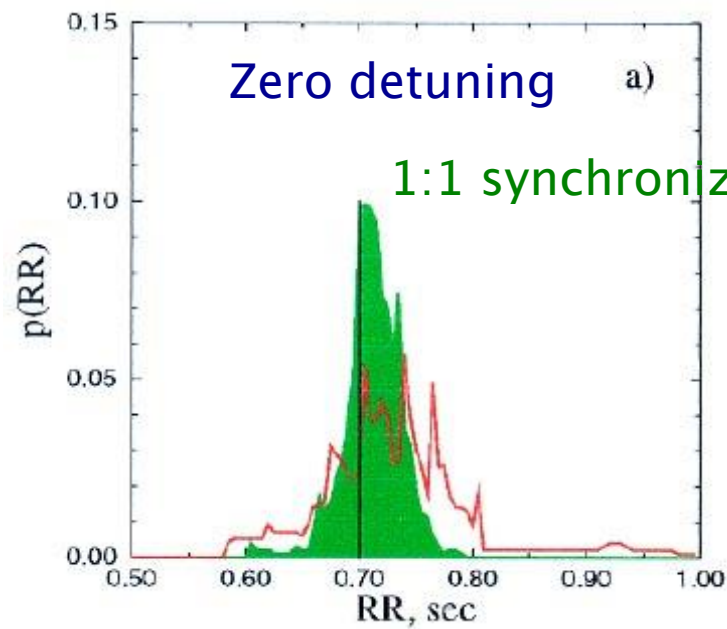
Non-periodic beeping  
Large detuning

Black – unforced, Green - forcing

Black – forced, Green - forcing



# Experimental results – RR distributions



Red – distribution of RR intervals without forcing

Green - distribution of RR intervals with forcing

Black line – frequency of forcing

# Previous studies with heart beats: 2

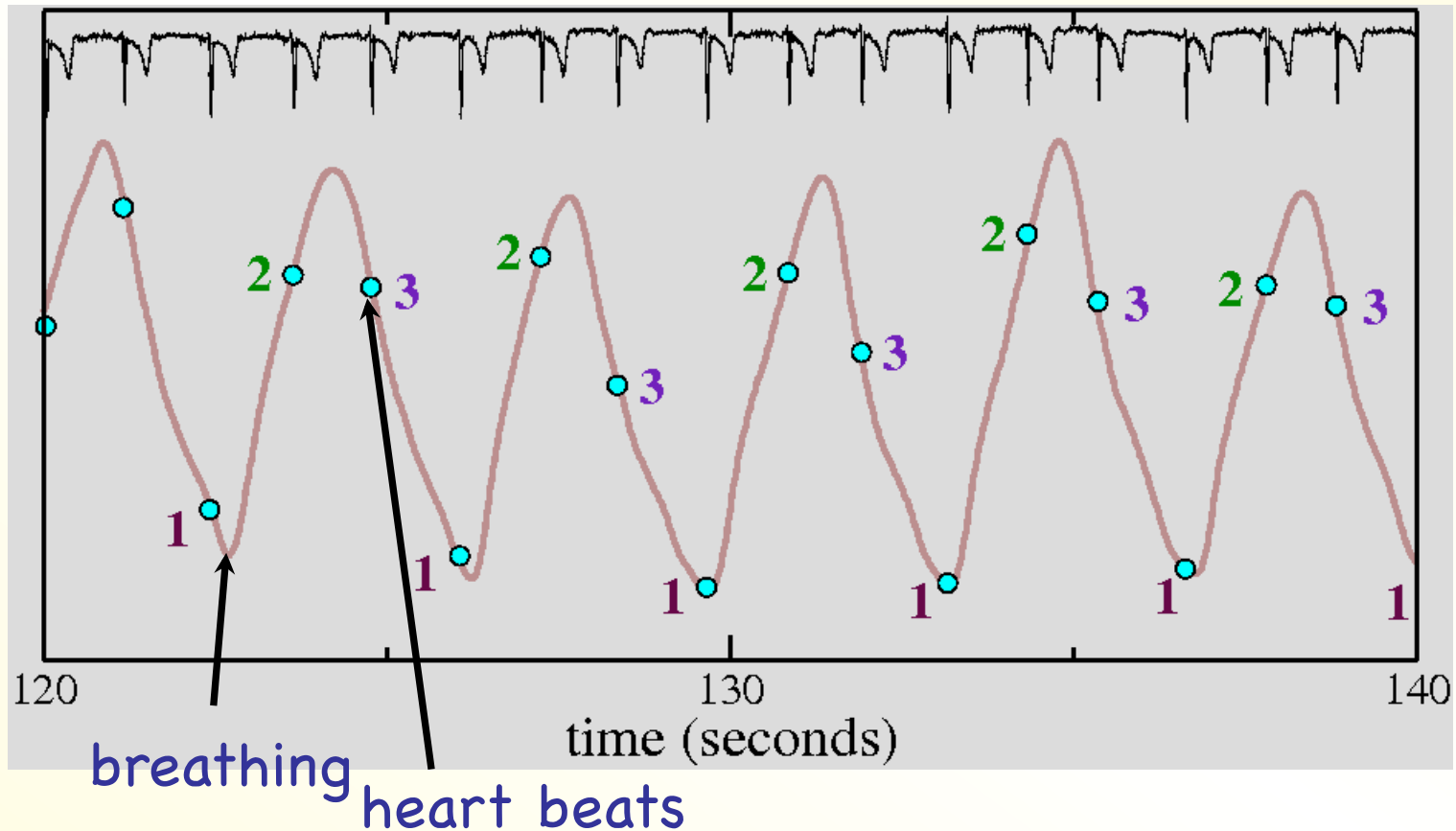
Spontaneous breathing can entrain heart beats.

[C.Schafer, M.G. Rosenblum, J. Kurths, H.-H. Abel, Nature 392, 239 (1998)]



# Previous studies with heart beats: 3

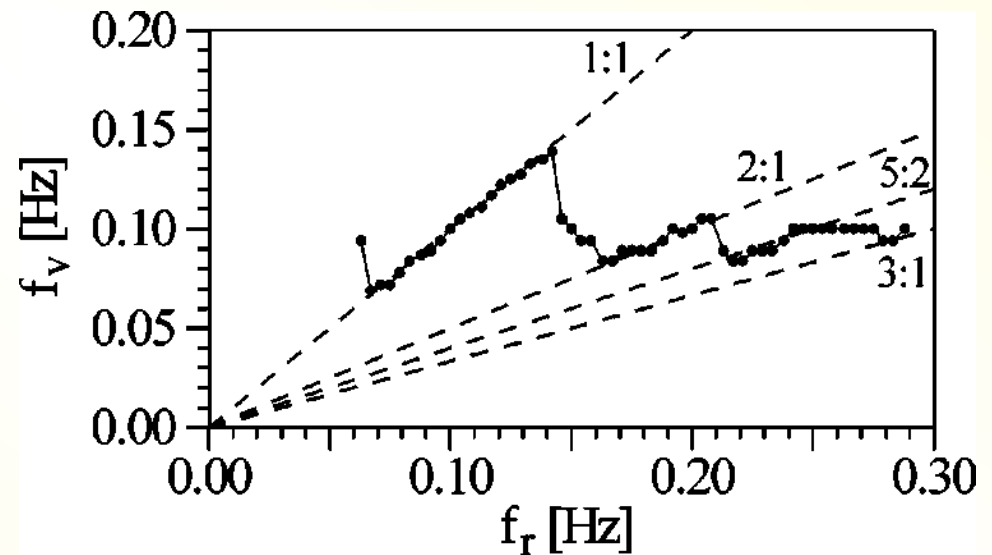
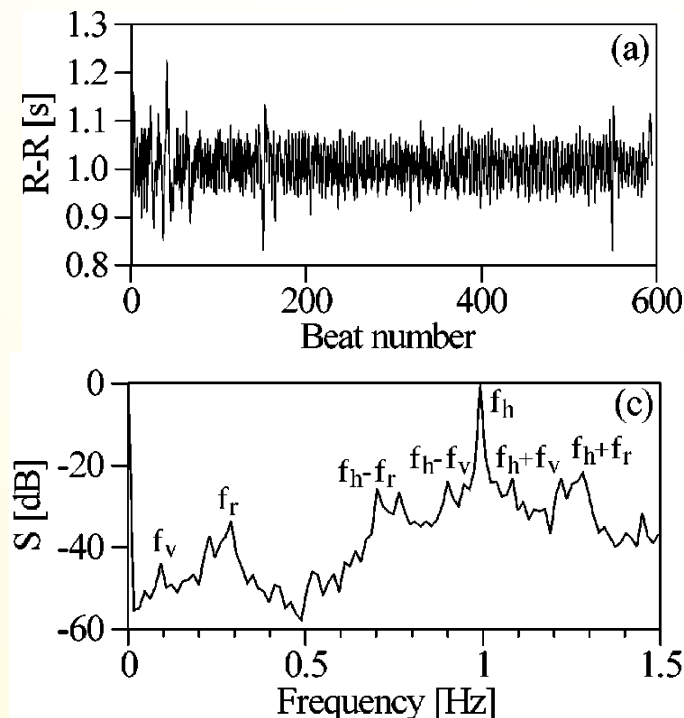
Paced breathing can entrain heart beats.



[S. Rzechiniński, A.G. Balanov, N.B. Janson, P.V.E. McClintock, Phys. Rev. E 66, 051909 (2002) ]

# Previous studies with heart beats: 4

Paced breathing can entrain slow component of heart beats. Breathing rate is monotonously increased during 30 minutes. Slow component of heart beats is entrained.



[M.D. Prokhorov, V.I. Ponomarenko, V.I. Gridnev, M.B. Bodrov, A.B. Bespyatov, Phys. Rev. E 68, 041913 (2003) ]

# What is delayed feedback

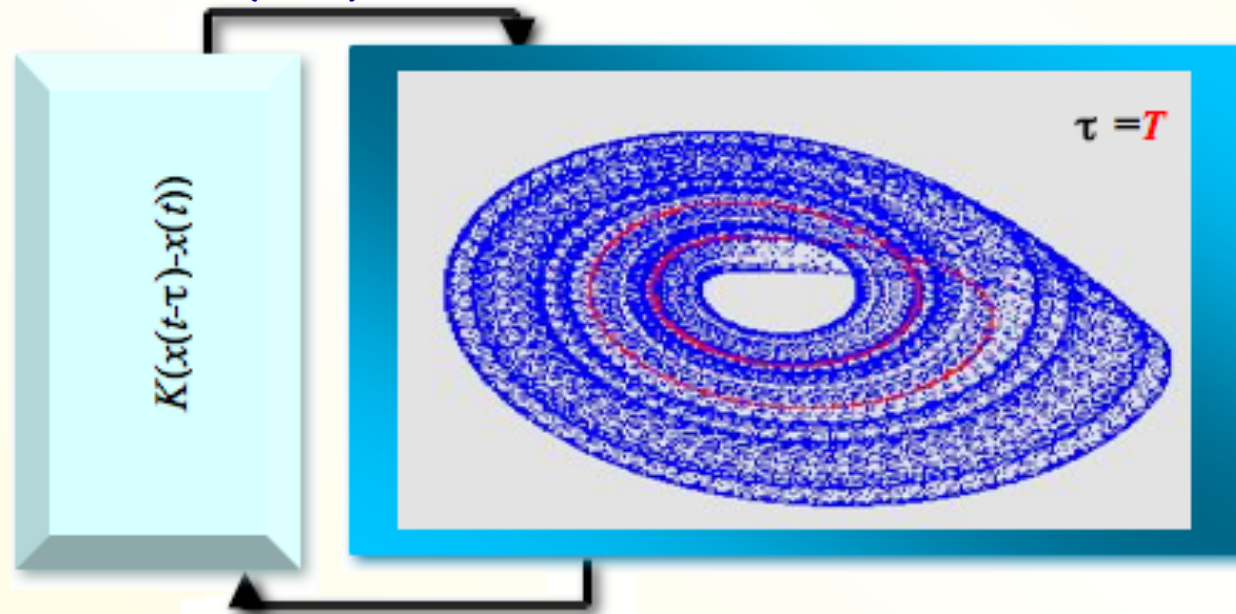
$$F(t) = K(s(t - \tau) - s(t))$$

$s(t)$  is some signal measured from the system in real time which can be any, possibly nonlinear, combination of system variables;

$K$  is the strength of the feedback

$\tau$  is the amount of the time delay

Originally introduced for deterministically chaotic systems and intended to stabilise unstable periodic orbits (UPO).



[K. Pyragas, Phys. Lett. A 170, 421 (1992) ]

[K. Pyragas, Phys. Lett. A 206, 323 (1995) ]

# How can delayed feedback affect self-oscillatory systems?

1. In deterministically chaotic systems:  
Convert chaotic oscillations into periodic.

1. In periodically oscillating systems:

a) Change period of oscillations

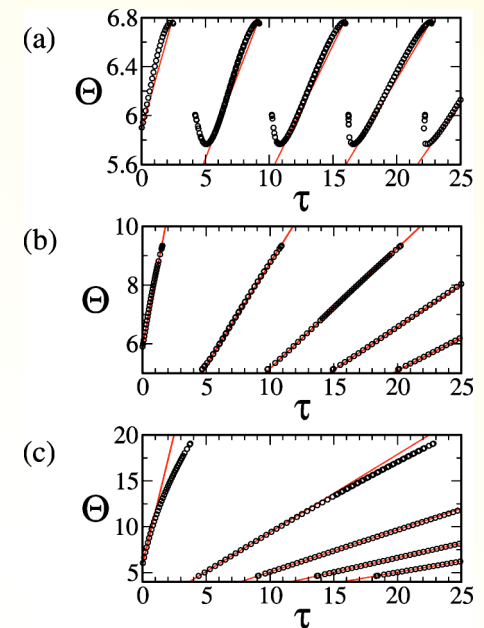
[W. Just et al, Phys. Rev. Lett. 81, 562 (1998) ]

[J. Xu and K.W. Chung, Physica D 180, 17 (2003) ]

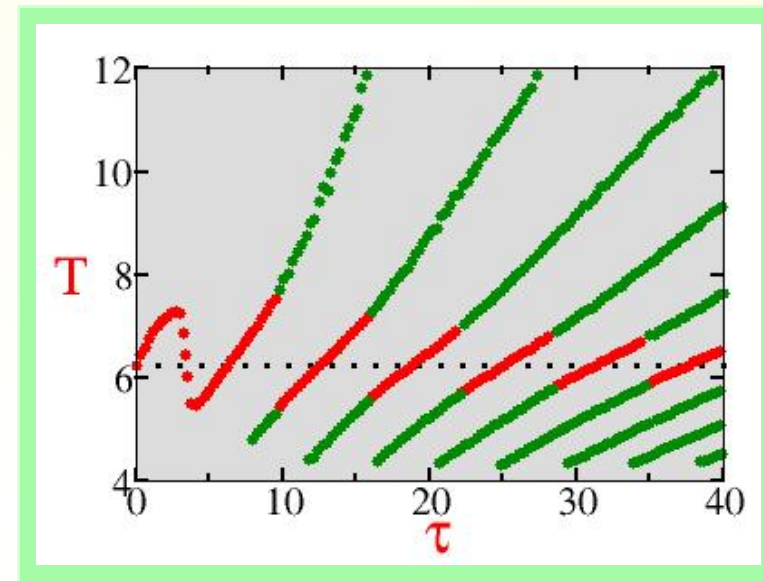
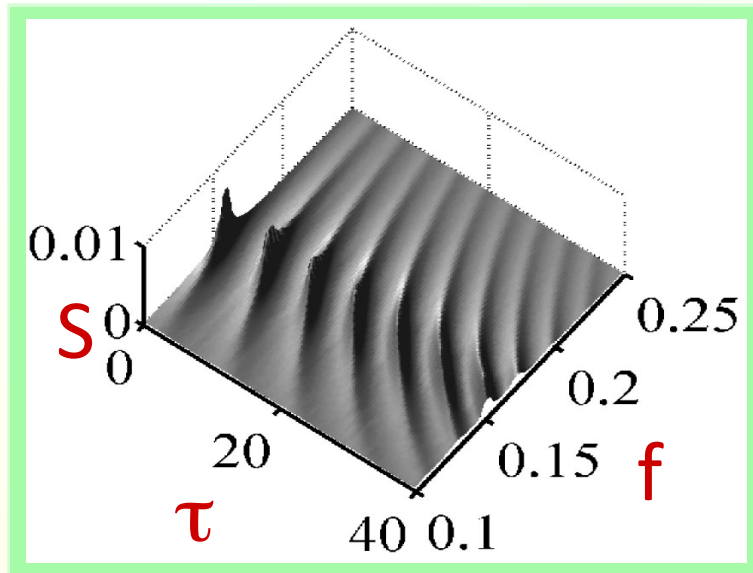
[A.G. Balanov, N.B. Janson, E. Scholl, Phys. Rev. E 71, 016222 (2005)]

b) Make oscillations chaotic

[J. Weiner, F.W. Schneider, K. Bar-Eli, J. Chem. Phys. 93, 2704 (1989) ]



# How can delayed feedback affect noise-induced oscillations?



Peak "period":  $T=1/f$

●●●● all spectrum peaks  
●●●● highest peak

[N.B. Janson, A.G. Balanov, E. Scholl, Phys. Rev. Lett. 93, 010601 (2004) ]

[A.G. Balanov, N.B. Janson, E. Scholl, Physica D 199, 1 (2004) ]

[E. Scholl, A.G. Balanov, N.B. Janson, A. Neiman, Stochastics and Dynamics 5, 281 (2005) ]

[N.B. Janson, A.G. Balanov, and E. Scholl, Control of noise-induced dynamics. Handbook of Chaos Control, 2<sup>nd</sup> Ed, E. Scholl, H.G. Schuster (Eds) 223 2007)]

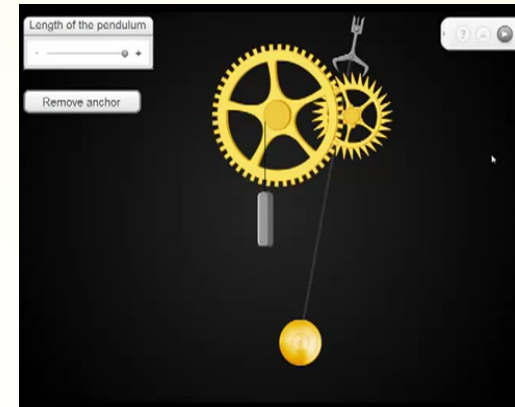


# What do we know about breathing

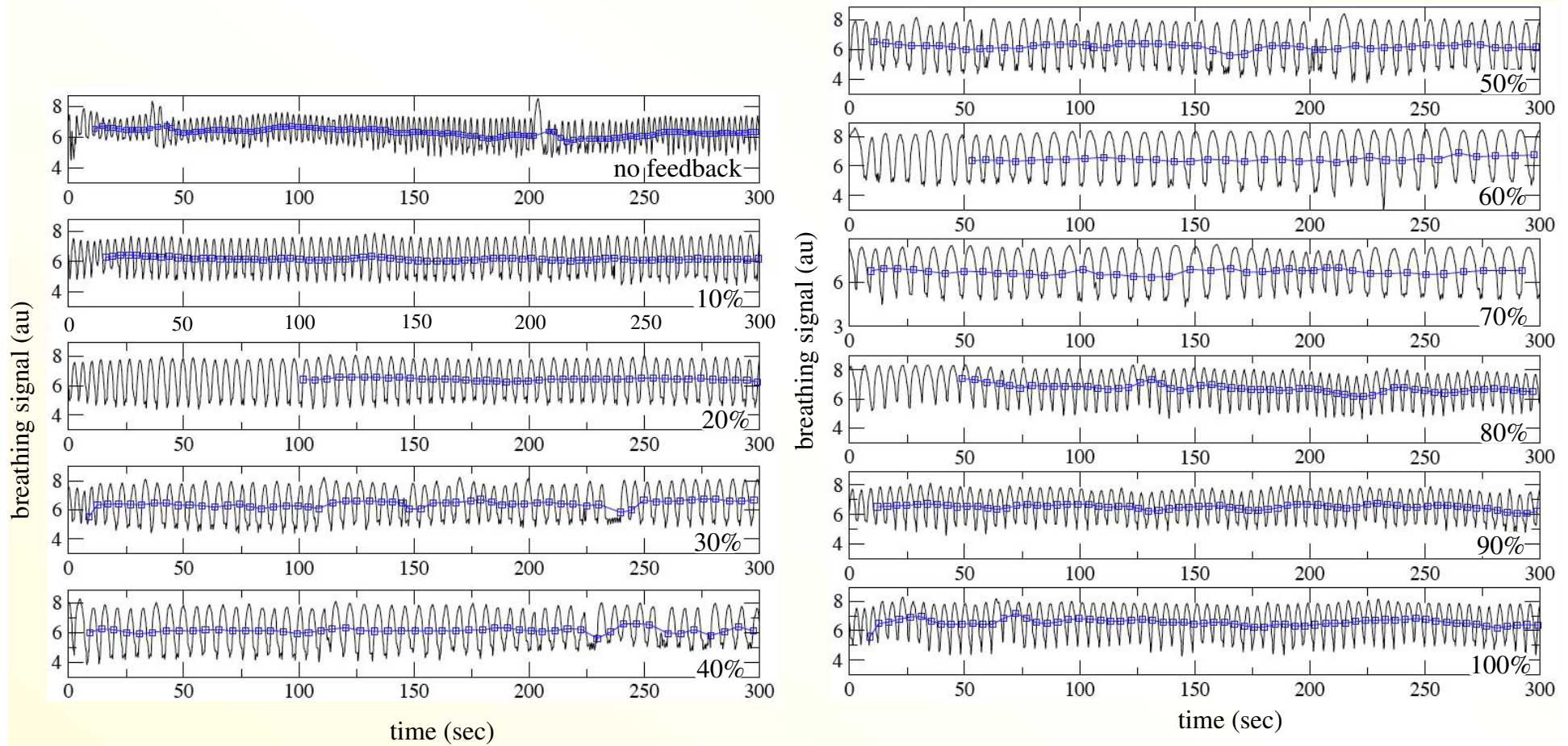
1. Never stops (while the subject lives)
2. Rhythmic, but non-periodic
3. Occurs in a non-linear dissipative system

## Assume

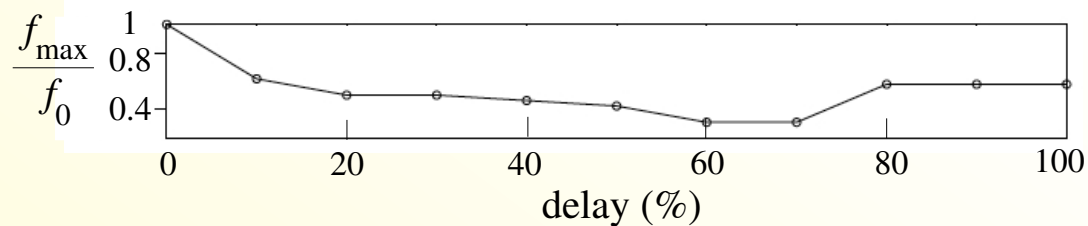
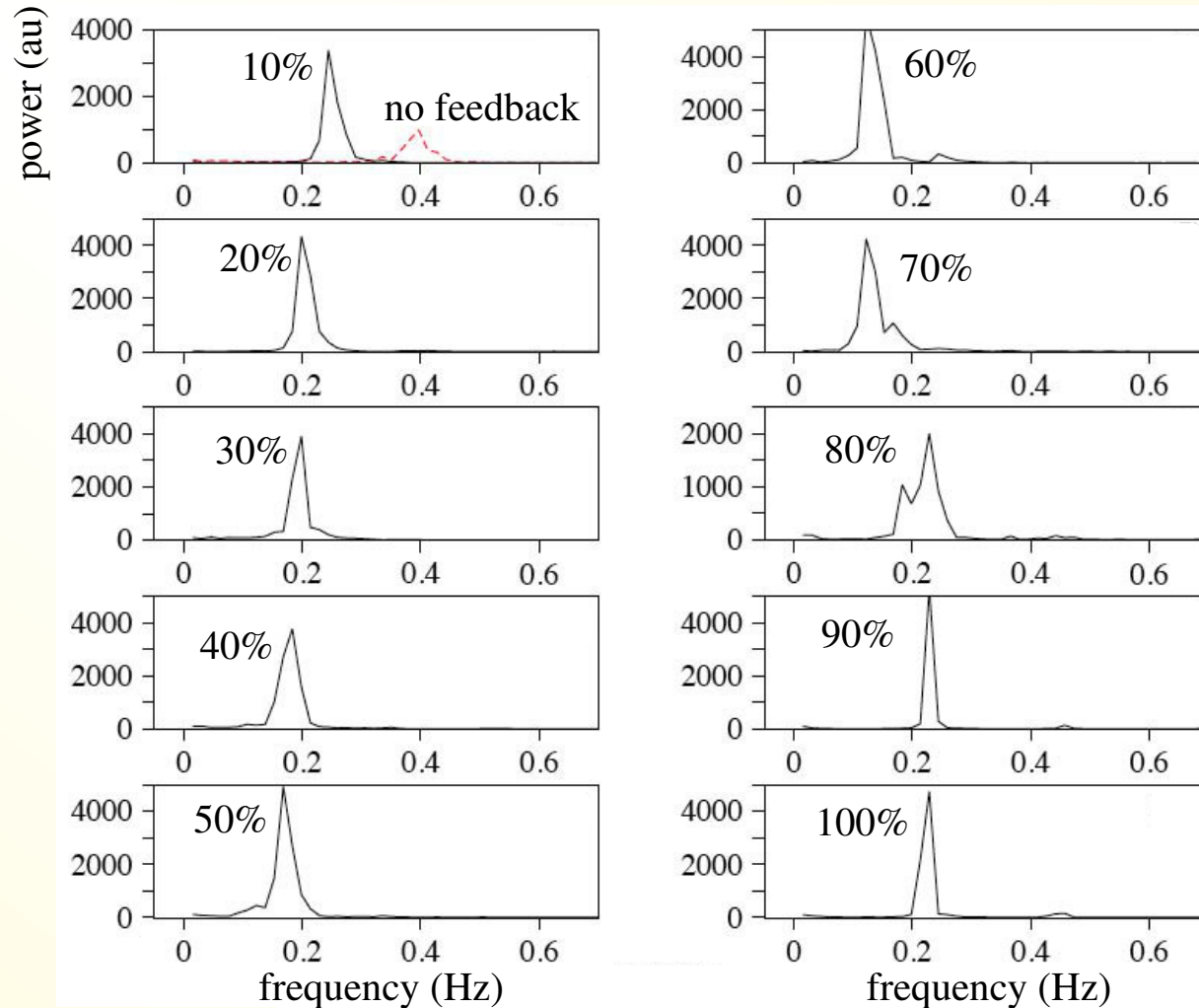
- Breathing is a self-oscillating process.
- Just like heart beats, but can be consciously controlled.
- Mathematical model: randomly perturbed limit cycle?



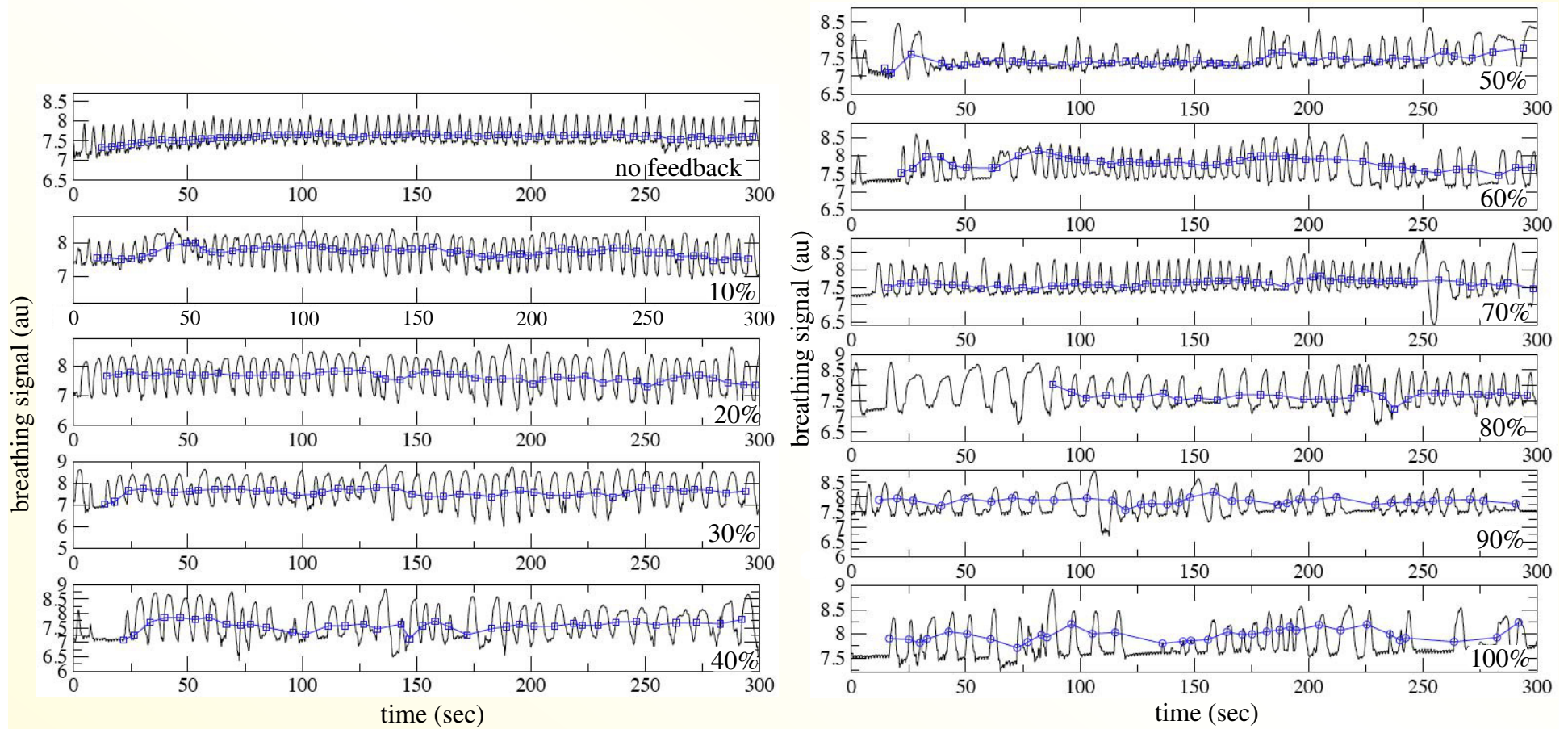
# Breathing slowed down: signals



# Breathing slowed down: parameters

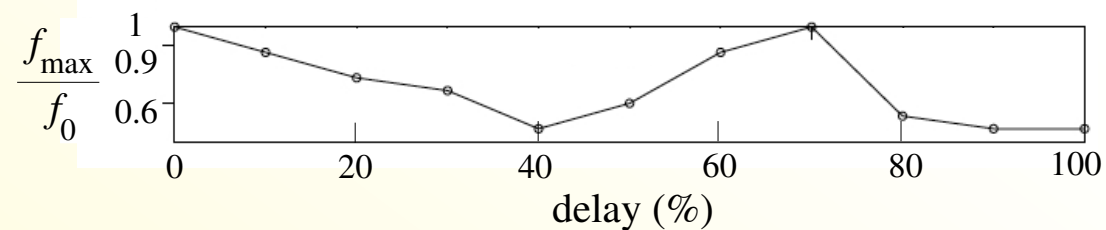
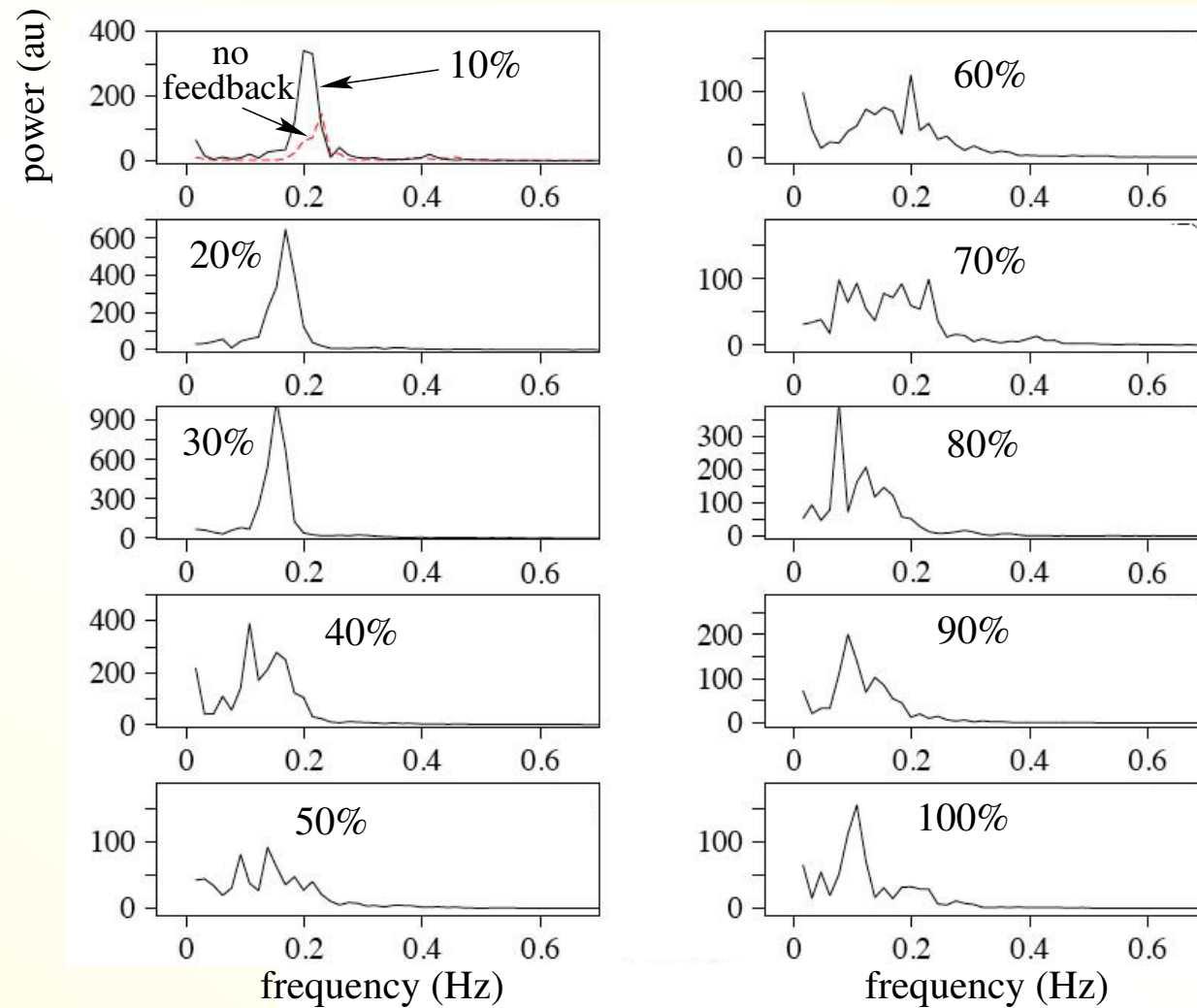


# Chaotisation of breathing: signals





# Chaotisation of breathing: parameters

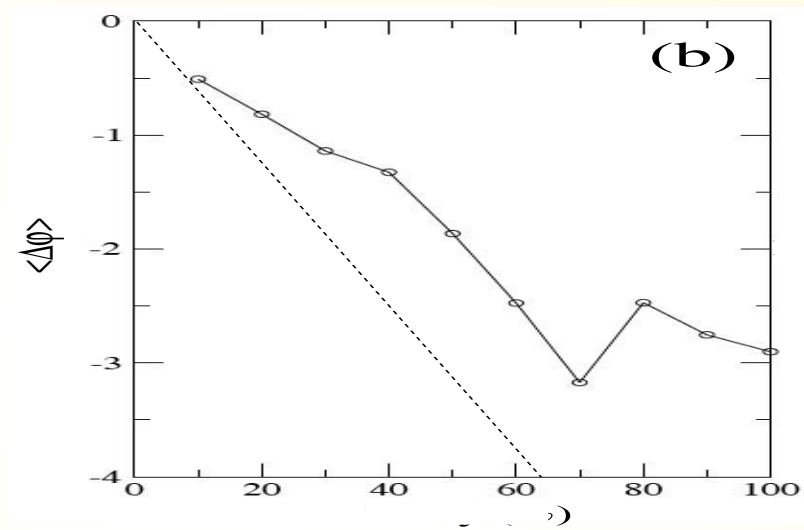
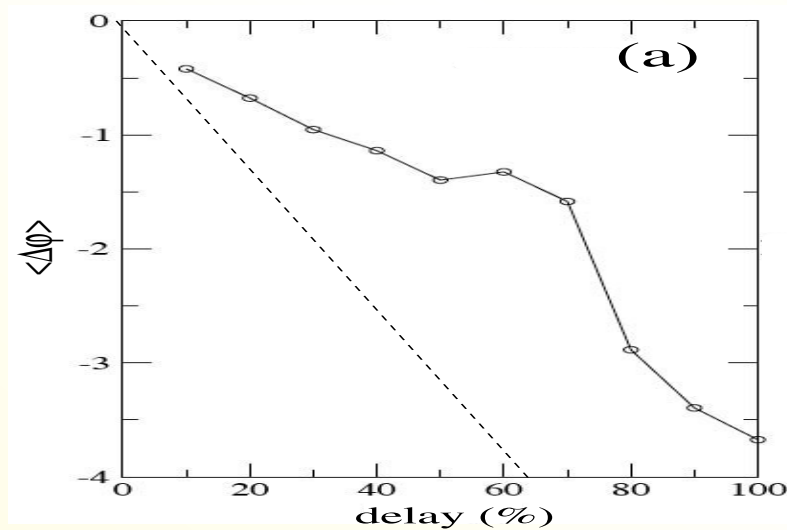




# Average phase difference with delayed feedback

Phase of breathing

$$\phi(t) = 2\pi(i - 1) + 2\pi \frac{t - t_i}{t_{i+1} - t_i}, \quad i = 1, 2, \dots$$



Dashed straight lines – if feedback had no effect on the breathing.

- (a) Breathing slows down between 10% and 70%, while staying regular for all delays.
- (b) Breathing slows down while staying regular for delays 10% to 30%, and becomes irregular at 40% and more.

# Summary

Out of 24 volunteers:

1. in 11 people breathing was slowed down as the percentage delay grew.

Within this group

a) in 6 humans breathing stayed quite regular

b) In 5 volunteers transition from periodic to irregular breathing pattern

2. In 3 volunteers breathing became faster with delay.

In this group,

a) in 2 humans breathing remained regular

b) in 1 human it became very irregular.

3. In 1 volunteer breathing became more regular with delayed feedback than without, but retained its average period.

4. 9 volunteers showed no apparent response to the delayed feedback.

[N.B. Janson, A. Pototsky, C. Parkes, Delayed feedback applied to breathing in humans, European Physical Journal 222(10) pp 2623-2631 (2013). ]

# Discussion

1. Slowing down for moderate values of delay is known to be typical – consistent.
2. Chaotisation by delay is typical -- consistent
3. Due to noise and non-stationarity, it is difficult to detect effects in living systems – only very robust effects are detectable

## Acknowledgements

Thanks to Grigory Bordyugov for writing software for experiments.

Thanks to EPSRC for support.