Lattice Boltzmann Unlimited

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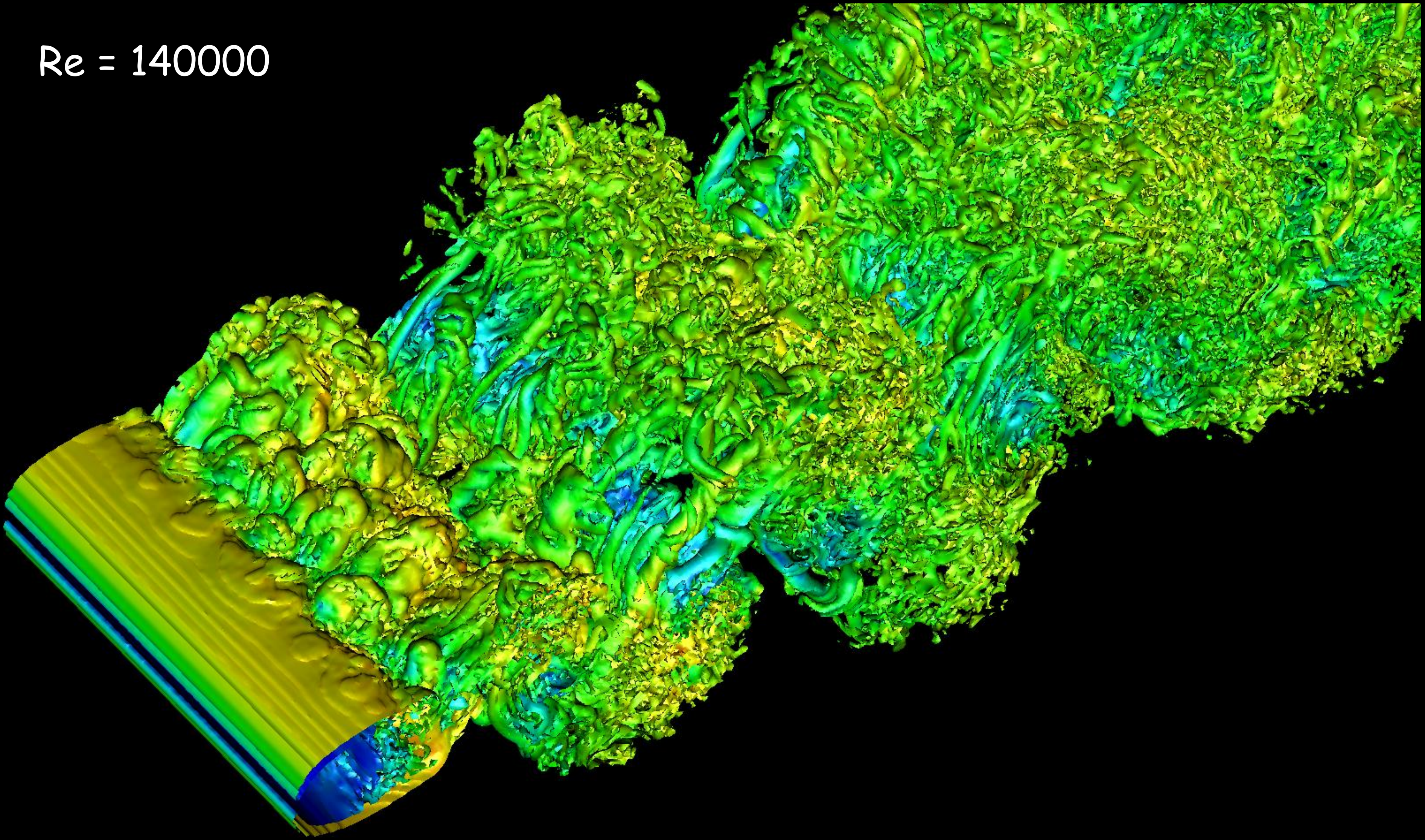
LAV

Direct numerical simulation at very high Reynolds numbers

Re = 140000



Karlin, Bösch & Chikatamarla, PRE (2014)


Let’s start …

Preliminaries

Boltzmann equation

$$\mathrm{Kn} = \frac{l_{\mathrm{m.f.p.}}}{L}$$

…scales out of the sum

Korteweg-type, not NS-type

Gorban & Karlin, PRL (1996)

Slemrod, Comp. Math. App. (2012)

Bösch & Karlin, PRL (2013)

Lattice Boltzmann equation

von Karman relation

$$\nu \sim l_{\mathrm{m.f.p.}} \times c_{\mathrm{s}}$$

$$\mathrm{Re} = \frac{L(U/c_{\mathrm{s}})}{\nu/c_{\mathrm{s}}} = \left(\frac{U}{c_{\mathrm{s}}}\right)\left(\frac{L}{l_{\mathrm{m.f.p.}}}\right) = \frac{\mathrm{Ma}}{\mathrm{Kn}}$$

Navier-Stokes equation

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} + \nabla p = \nu \nabla \cdot \nabla \boldsymbol{u}$$

Reynolds number

$$\mathrm{Re} = \frac{LU}{\nu}$$

Hard problem:

Singularity of energy dissipation

$$\dot{\varepsilon} \sim \nu \int |\nabla u|^2 d\boldsymbol{x} \sim O(1)$$

Lattice Boltzmann Method (LBGK)

Frisch, Hasslaher, Pomeau, Jimenez, Higuera, Succi, Benzi, Chen, d'Humieres … 1986-1999

$$f_i(\boldsymbol{x}+\boldsymbol{v}_i, t+1) - f_i(\boldsymbol{x}, t) = 2\beta(f_i^{\mathrm{eq}} - f_i)$$

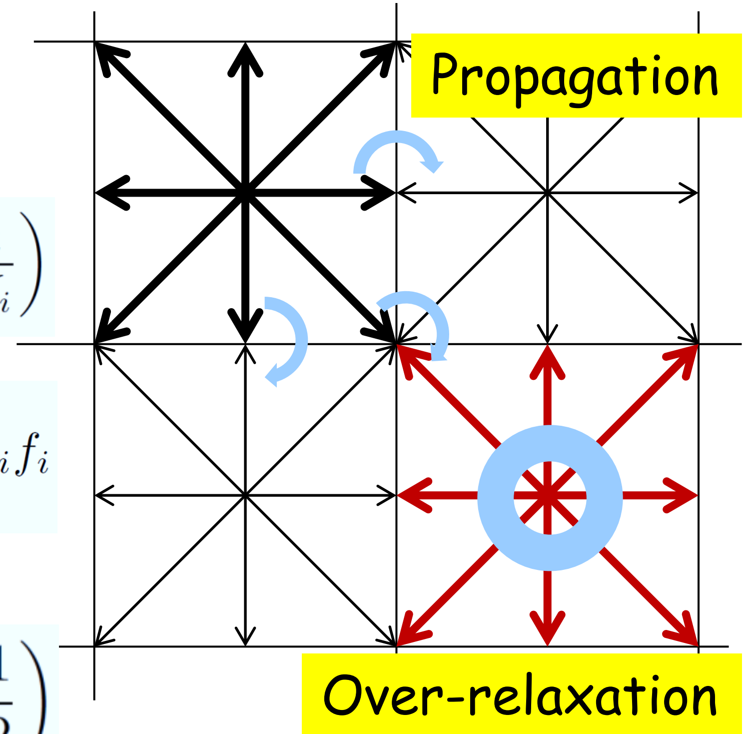
- Full discrete: **Velocity-Time-Space**
- Propagation: **Linear and Exact**
- Non-linearity: **Local Equilibrium**

*Minimizer of **entropy function***

$$H = \sum_{i=1}^{n} f_i \ln\left(\frac{f_i}{W_i}\right)$$

*under fixed **density** and **momentum***

$$\rho = \sum_{i=1}^{n} f_i, \ \rho\boldsymbol{u} = \sum_{i=1}^{n} \boldsymbol{v}_i f_i$$



- ***Low*** Mach number, ***high*** Reynolds number
- Recovers Navier-Stokes flow

Kinematic viscosity

$$\nu = c_{\mathrm{s}}^2\left(\frac{1}{2\beta} - \frac{1}{2}\right)$$

- ***Over-relaxation:*** High Reynolds numbers $\beta \to 1$
- ***Over-relaxation:*** LBM only (**not** from the Boltzmann equation)

cf: *Bösch & Karlin, Phys. Rev. Lett. (2013)*

Resolved simulation runs but…
Failure at sub-grid scale dynamics; No-go for high Reynolds numbers

Entopic Lattice Boltzmann Method

$$f_i(\boldsymbol{x} + \boldsymbol{v}_i, t + 1) - f_i(\boldsymbol{x}, t) = \alpha\beta(f_i^{\mathrm{eq}} - f_i)$$

- **Equilibrium:** Minimizer of entropy function… $H = \sum_{i=1}^{n} f_i \ln\left(\frac{f_i}{W_i}\right)$ … as before, **but**:
- ***Securing Second Law***

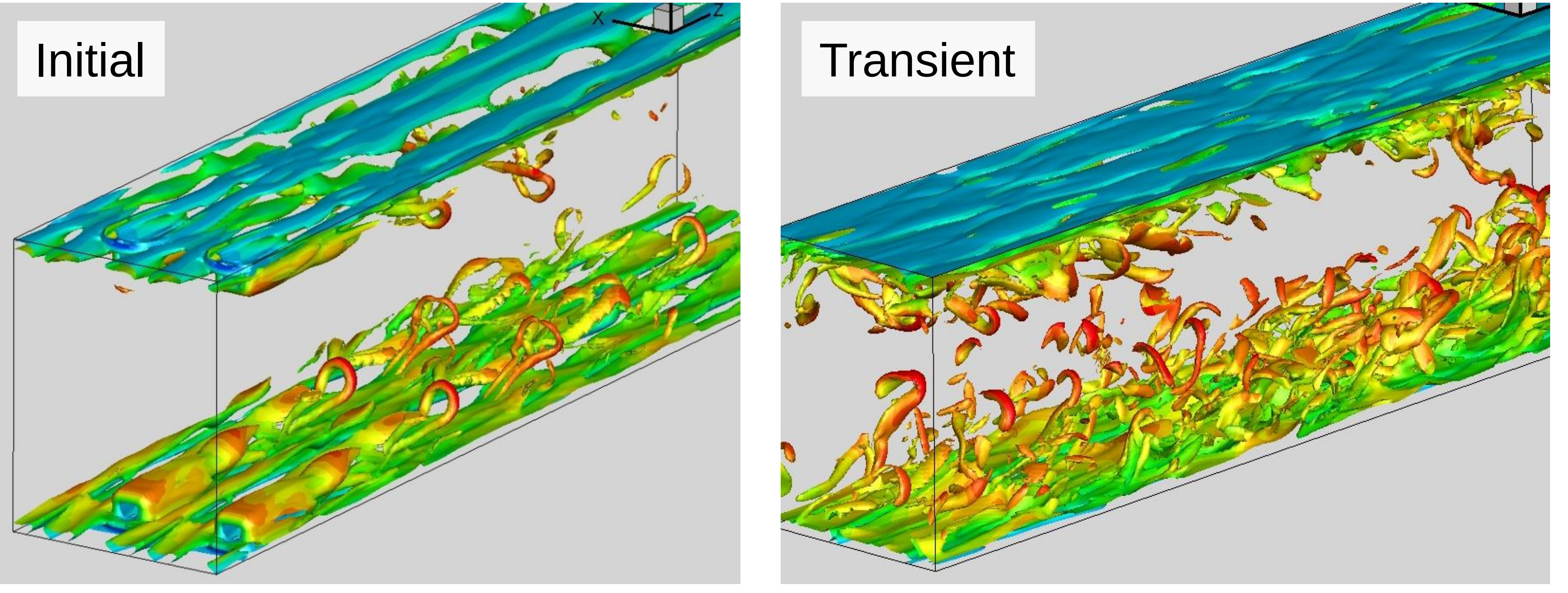
  Over-relaxation is computed at every node/time step (***key***)

$$H(f + \alpha(f^{\mathrm{eq}} - f)) = H(f)$$

- Unconditionally stable in the over-relaxation sector $\beta \to 1$
- Resolved limit *ELBM=LBM* : $\alpha = 2$
- Rescues the high Reynolds number regime

Karlin et al, ***Phys. Rev. Lett.*** *(1998)*; *Karlin et al,* ***Europhys. Lett.*** *(1999);*
Ansumali et al, ***Europhys. Lett.*** *(2003)*
Chikatamarla & Karlin, ***Phys. Rev. Lett.*** *(2006)* – complete classification of lattices

ELBM validation I: Turbulent channel flow



Ansumali et al, ***Phys. Rev. Lett.*** *(2006);*
Chikatamarla et al, ***J. Fluid Mech.*** *(2010);*
Karlin et al, ***Phys. Rev. E*** *(2011);*
Chikatamarla & Karlin, ***Physica A*** *(2013)*
Karlin et ai, ***Phys. Rev. E*** *(2013)*

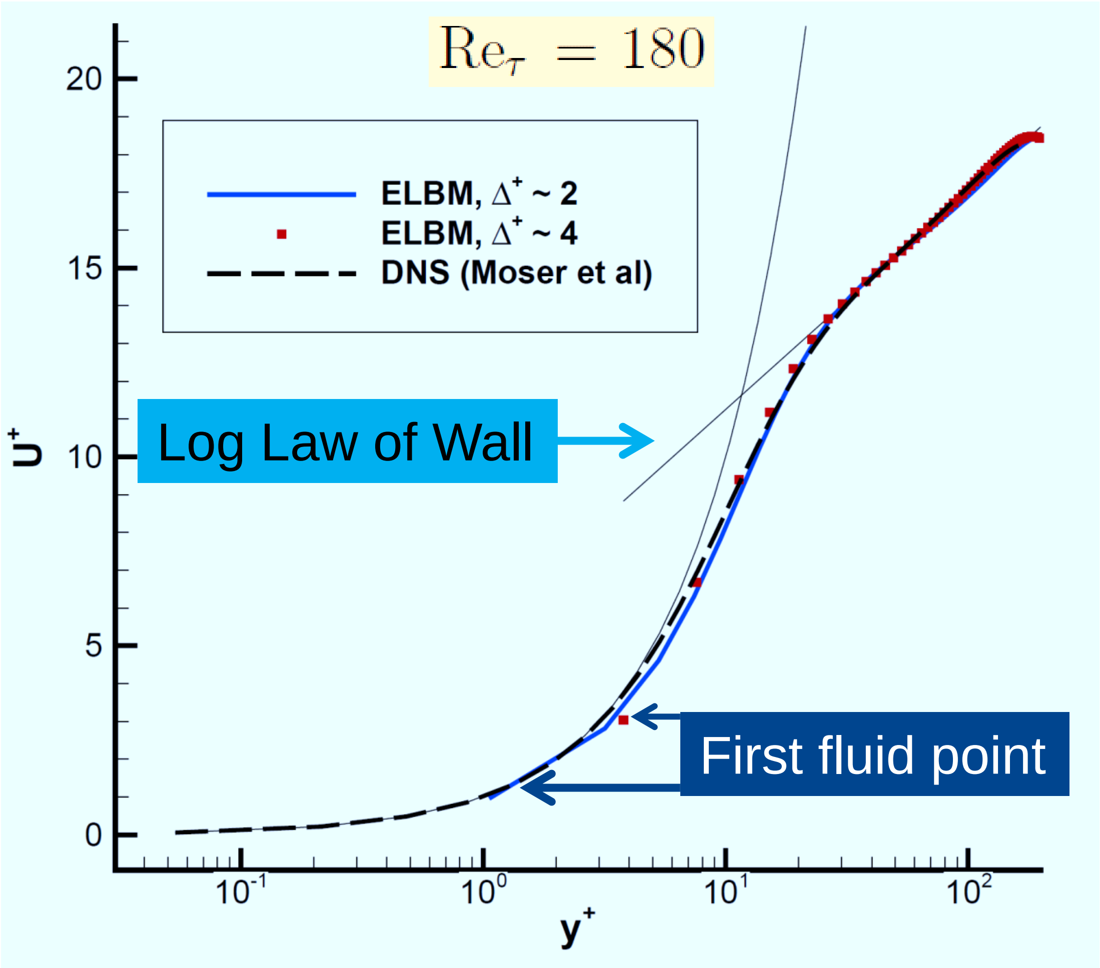
Turbulent channel flow: ELBM vs. DNS

Wall units: $y^+ = (u_\tau/\nu)y$

Resolution: $\Delta^+ = (u_\tau/\nu)\Delta$

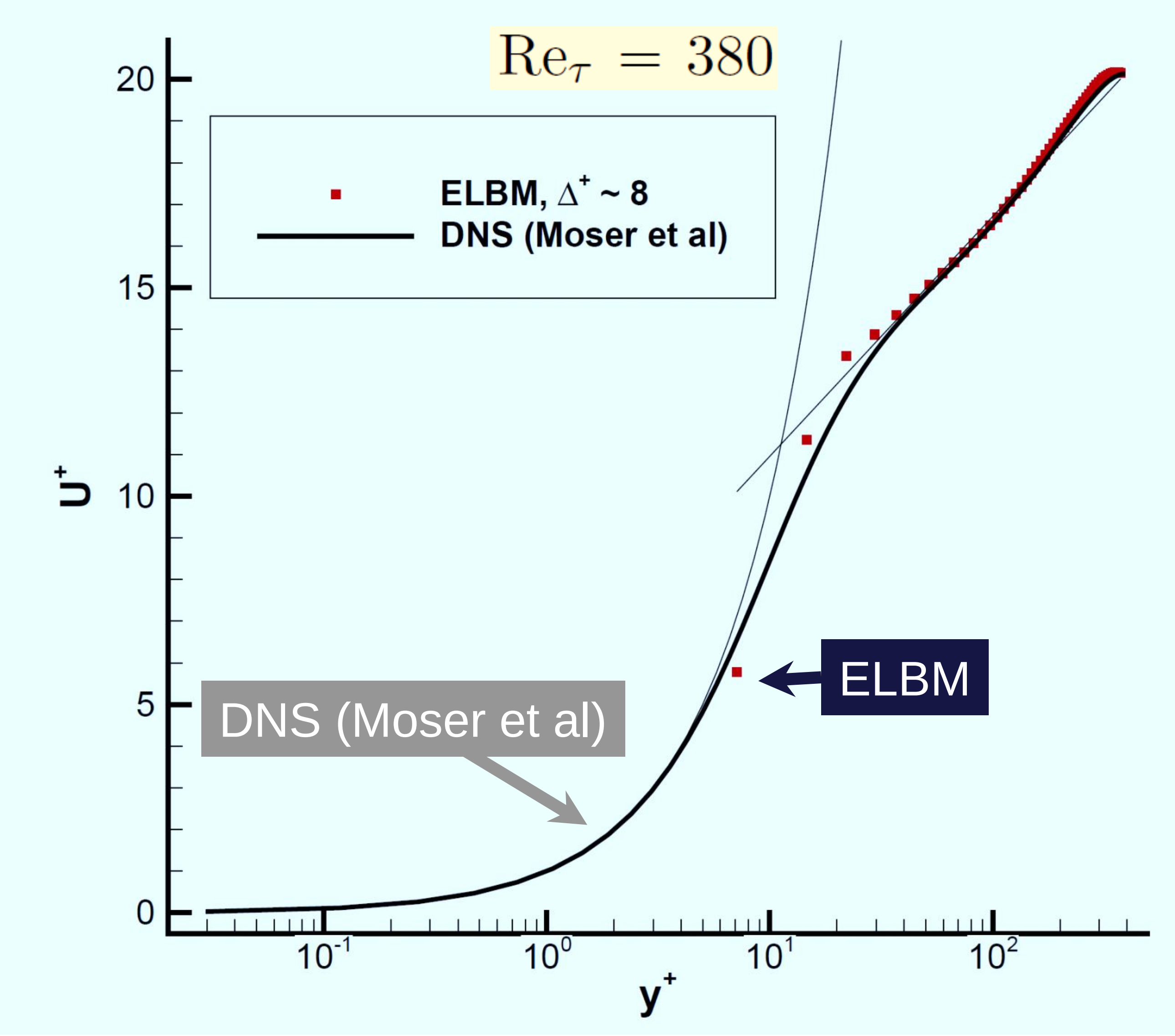
$$\mathrm{Re}_\tau = (u_\tau/\nu)H$$

Average velocity



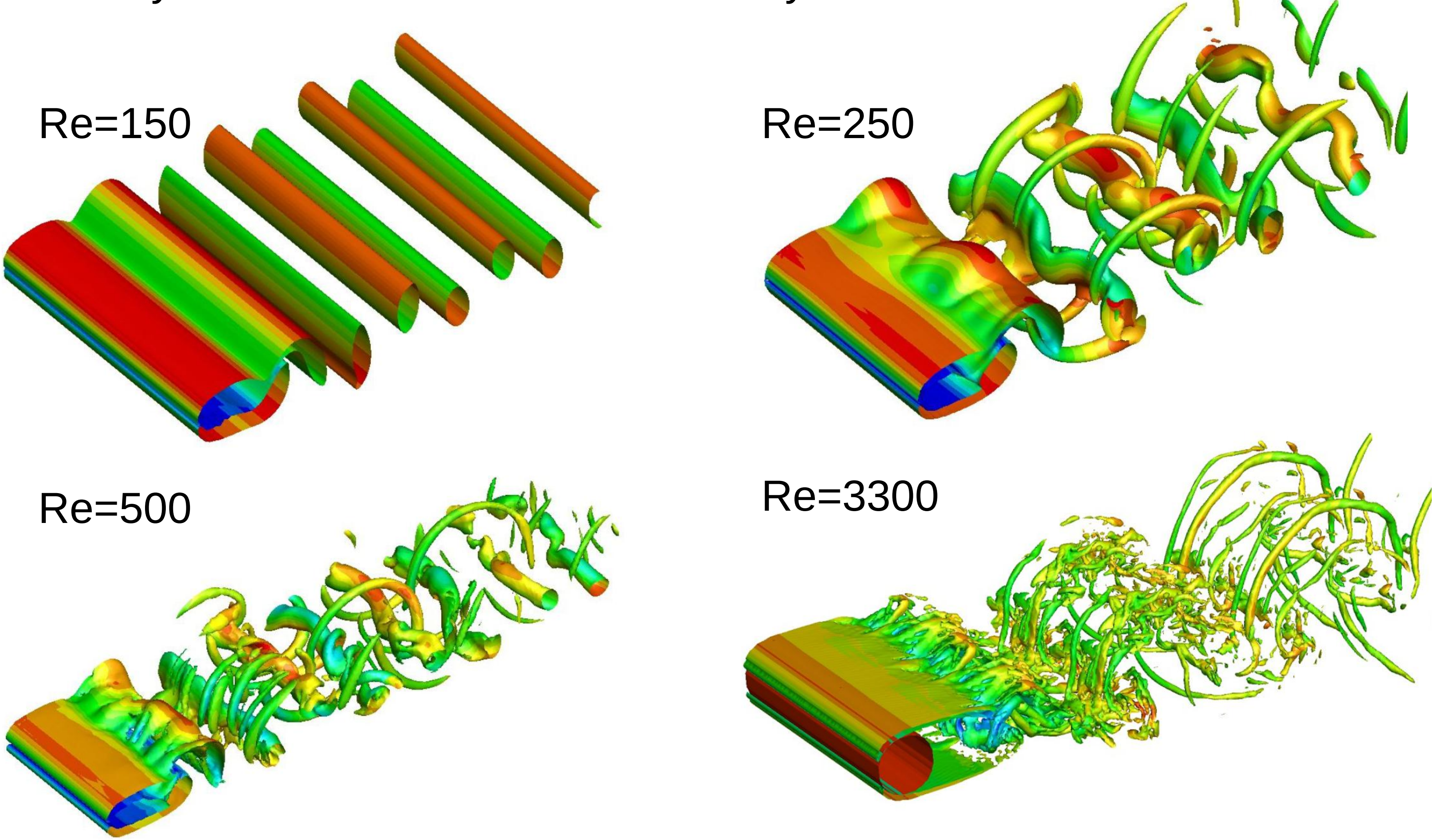
Turbulent channel flow (sub-grid)

Average velocity



ELBM validation II: Flow past a circular cylinder

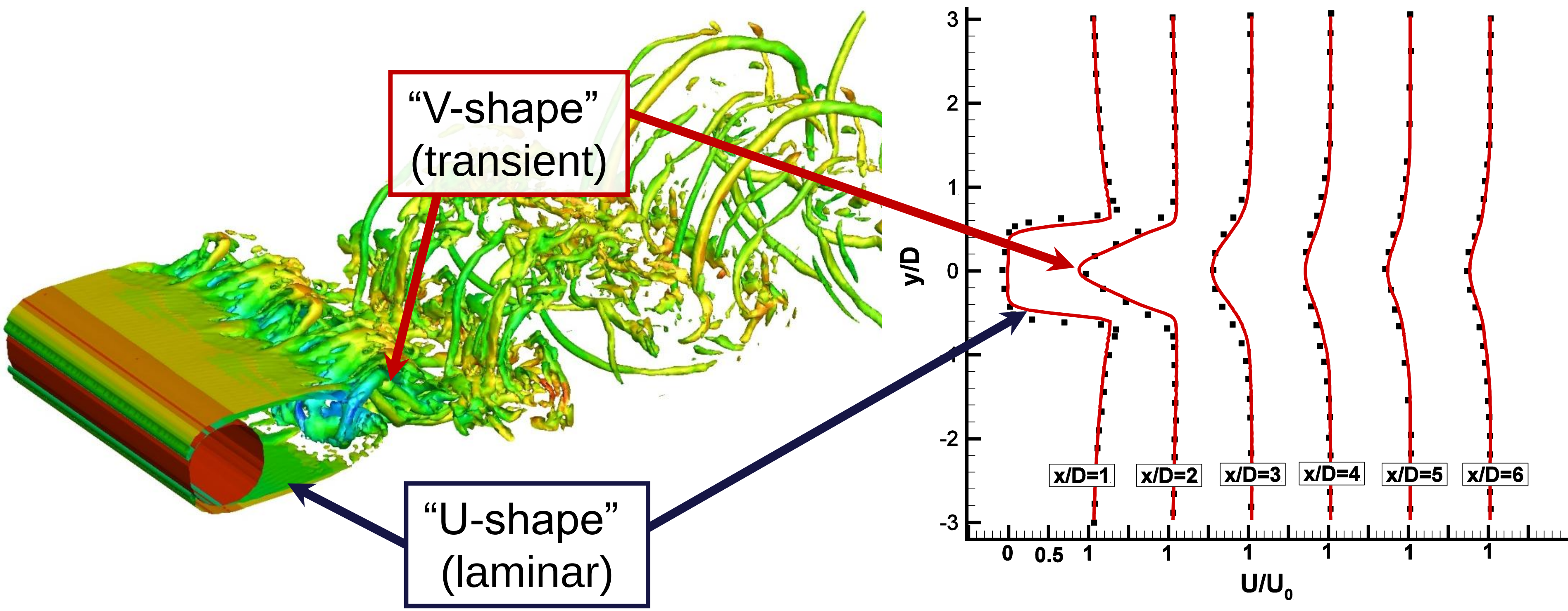
Vorticity iso-surfaces colored with velocity magnitude for various Reynolds numbers. As the Reynolds number is increased, transition to 3D and eventually transition to turbulence is clearly seen.



Flow past a circular cylinder

Re=3300

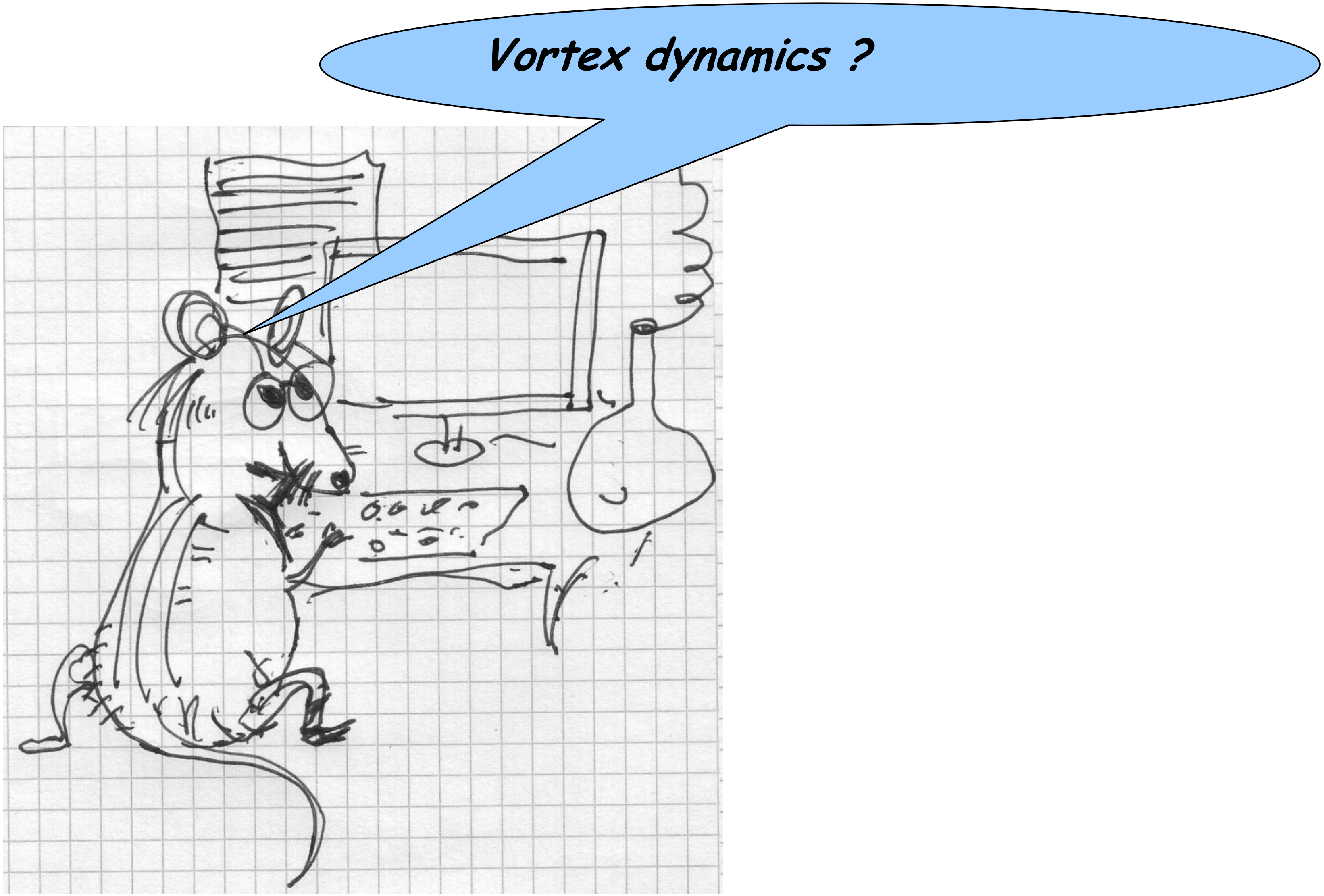
Average stream-wise velocity



Line: ELBM

Symbol: DNS, *Wissink & Rodi (2008)*

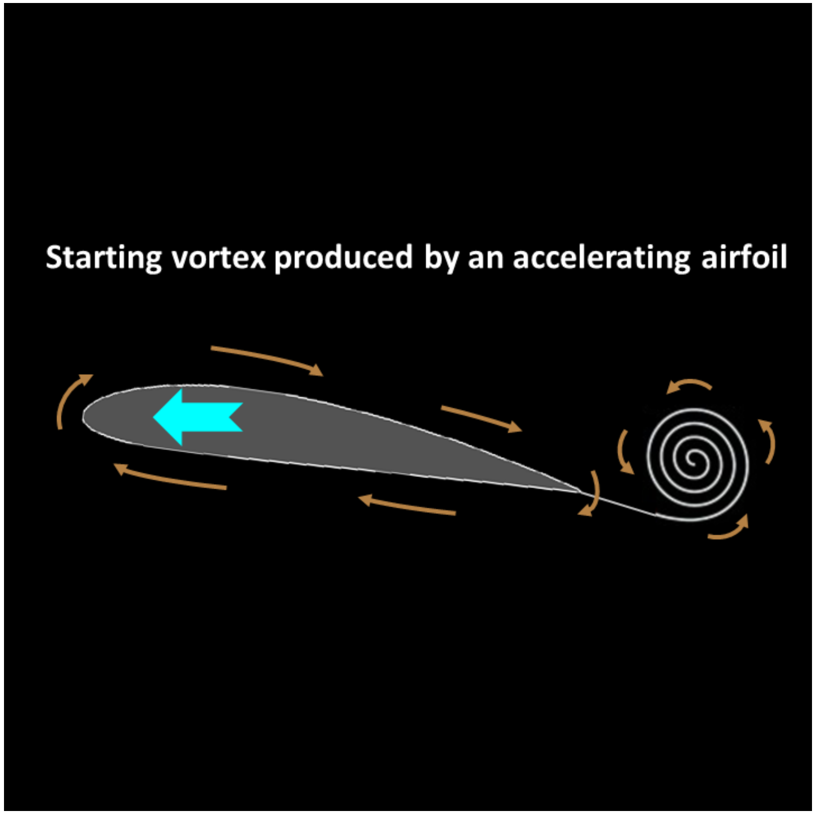
Resolution: Nodes ELBM ca. 1/10 Nodes DNS

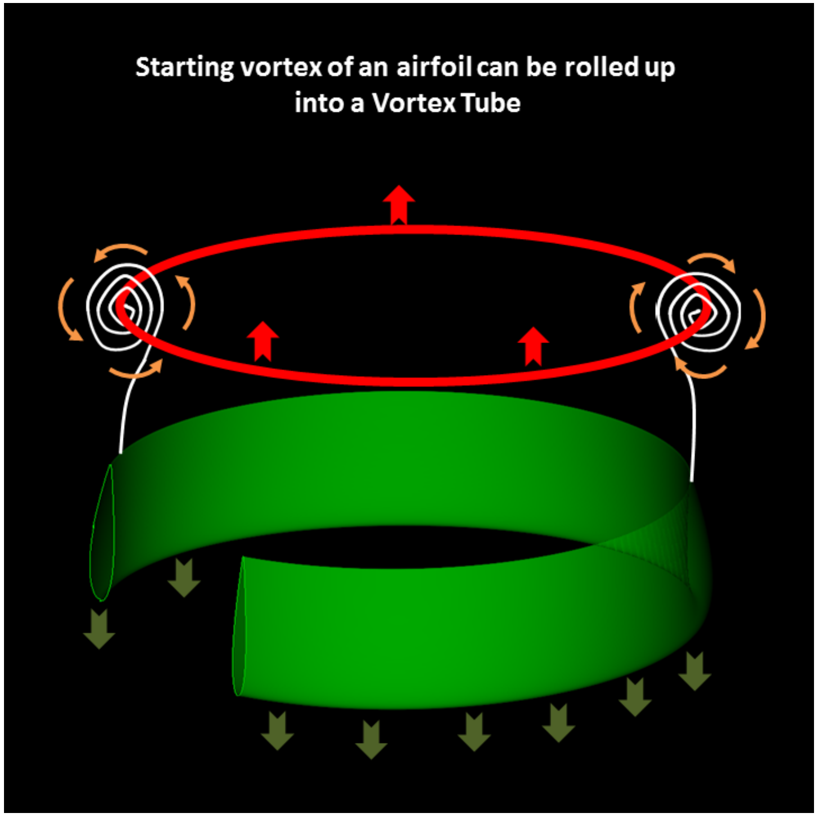


http://arxiv.org/abs/1310.3433

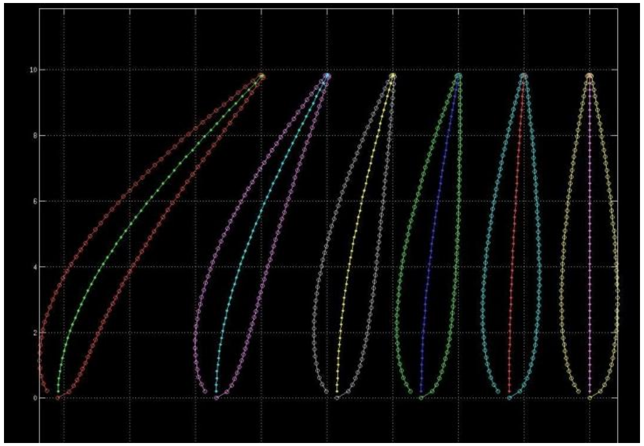
Knotted vortices I

- linked rings using special hydrofoils
- experimental setup of **Kleckner and Irvine, *Nature Physics* (2013)**
- Reynolds number from Re=**5000** to Re=**60000**
- grid size ca. **1000^3**
- computational time ca. **8 hrs on 2042 CPUs**

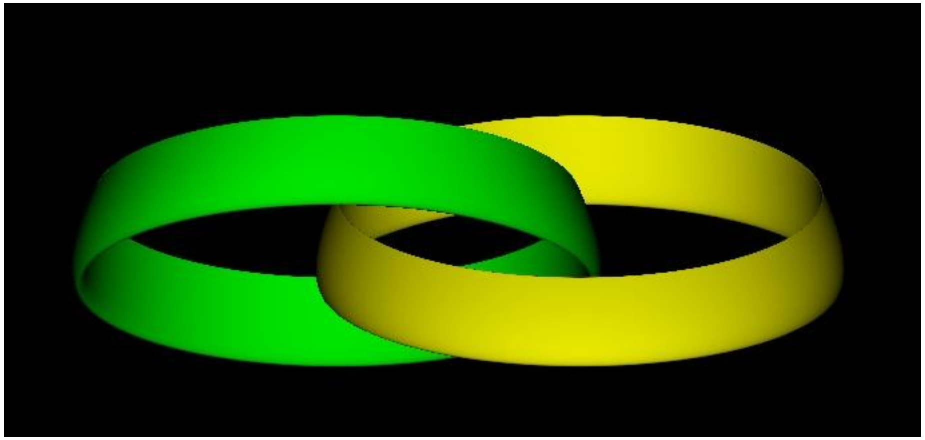




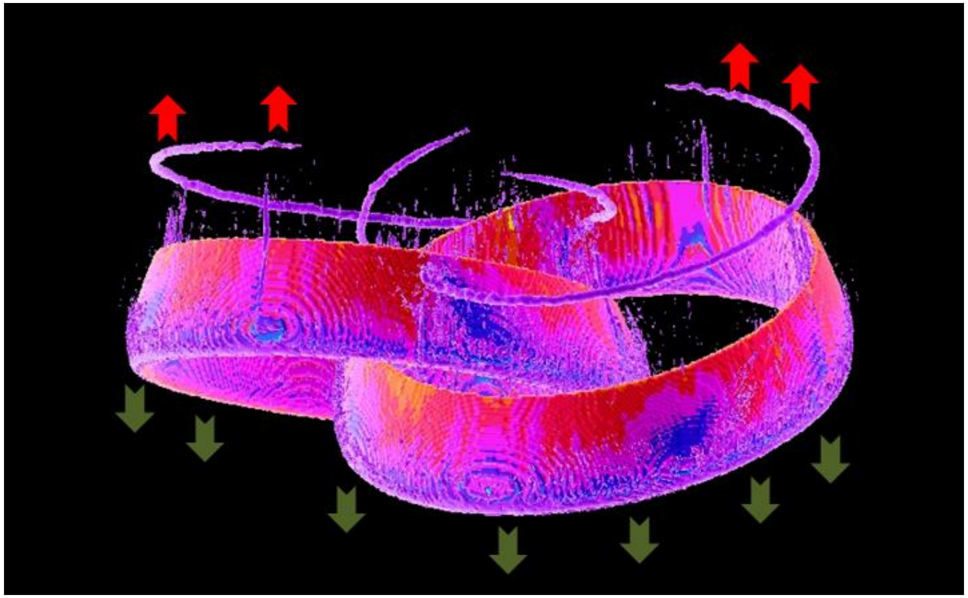
Knotted vortices II



Bent NACA profile

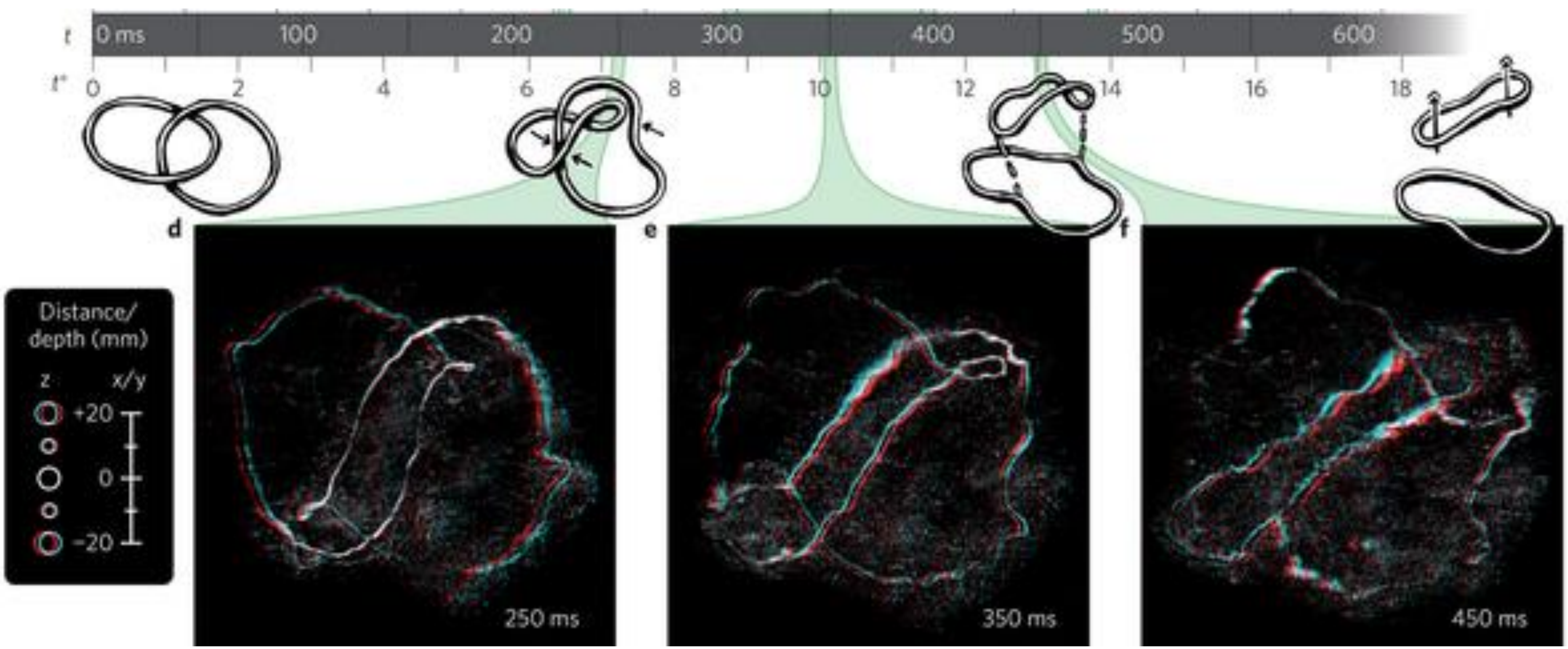


Linked rings 3D model



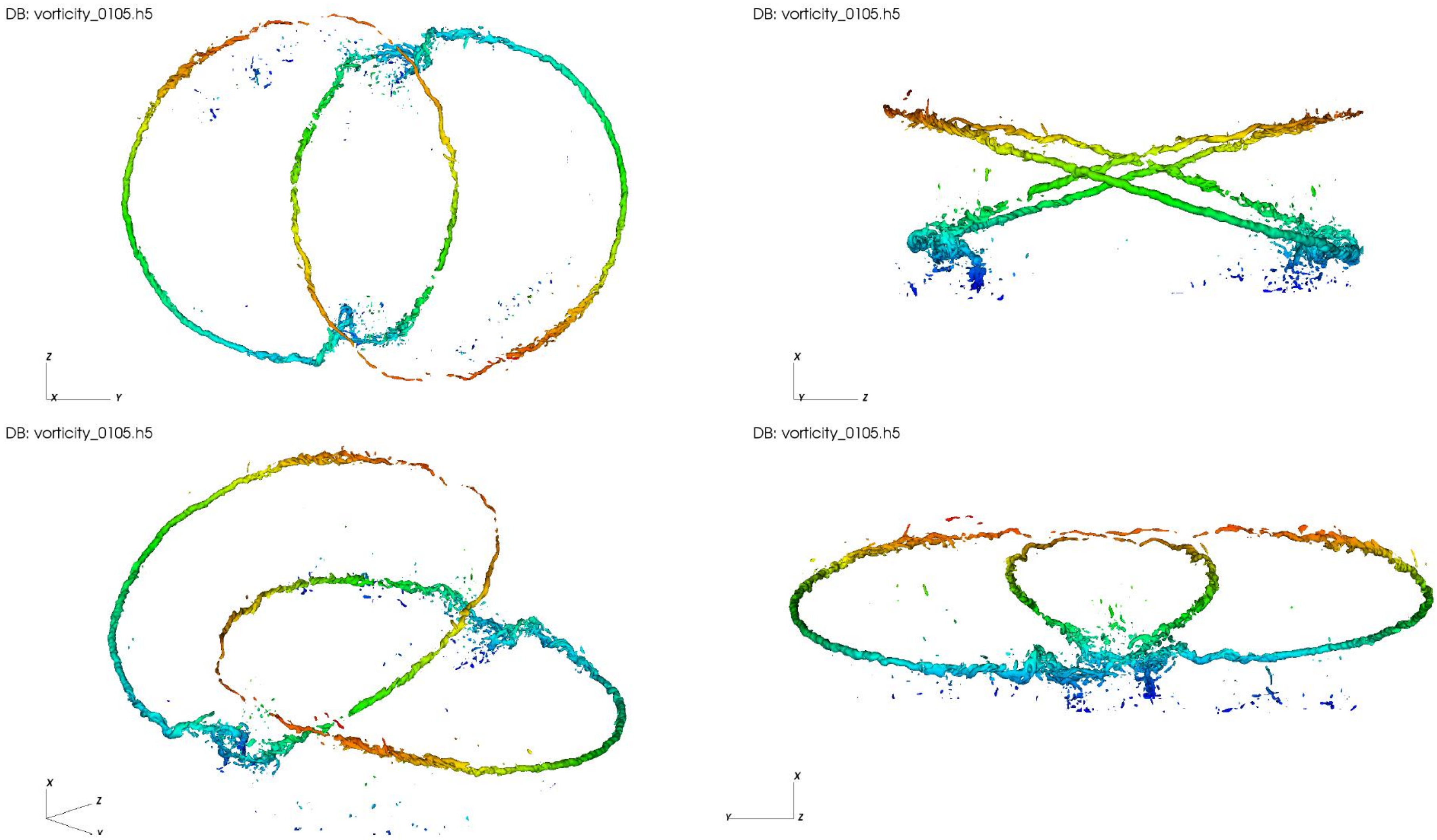
Initial stage

Experiment
Vortex reconnection

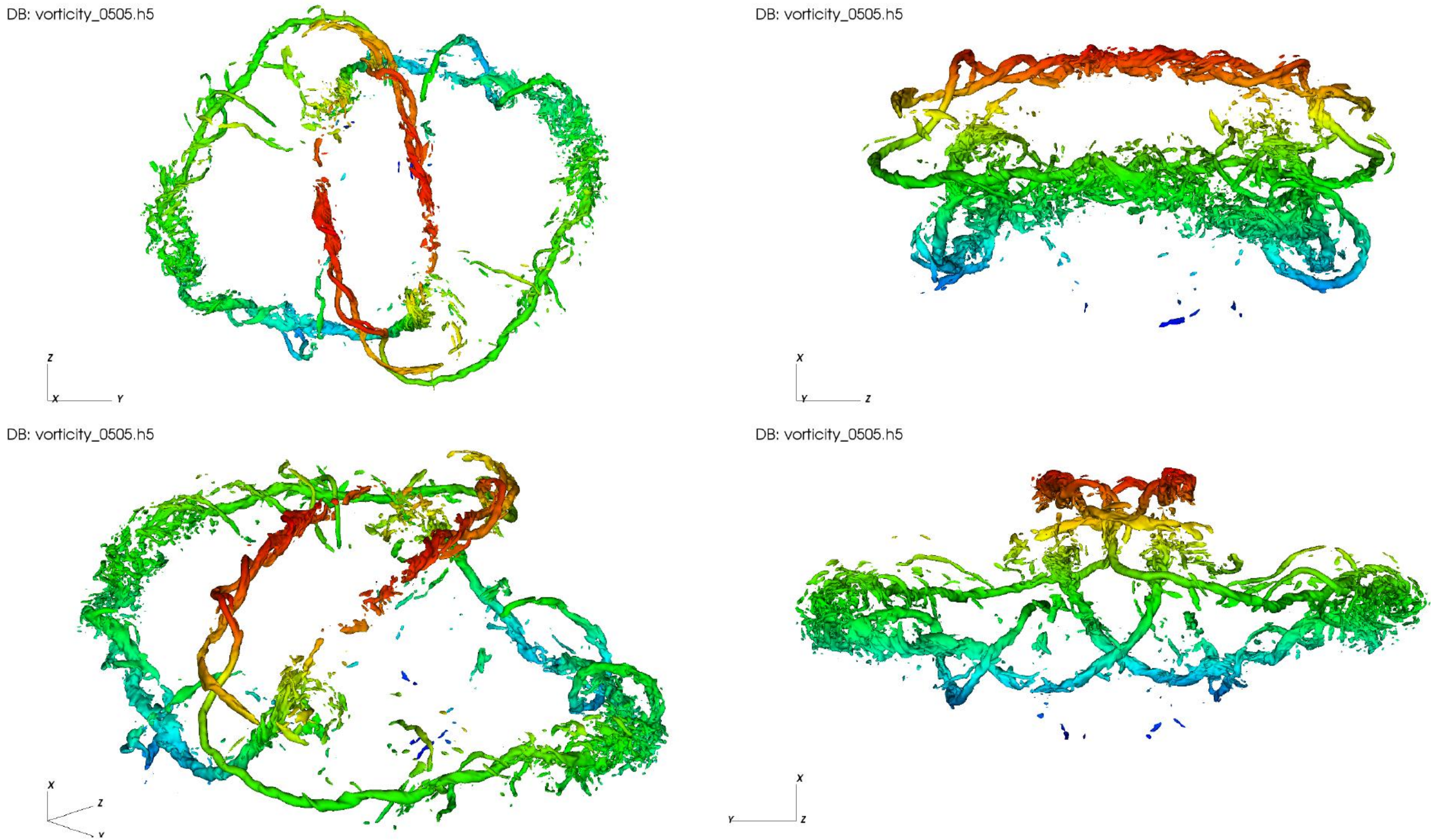


D. Kleckner and W. Irvine, *Nature Physics* **9**, 253–258 (2013)

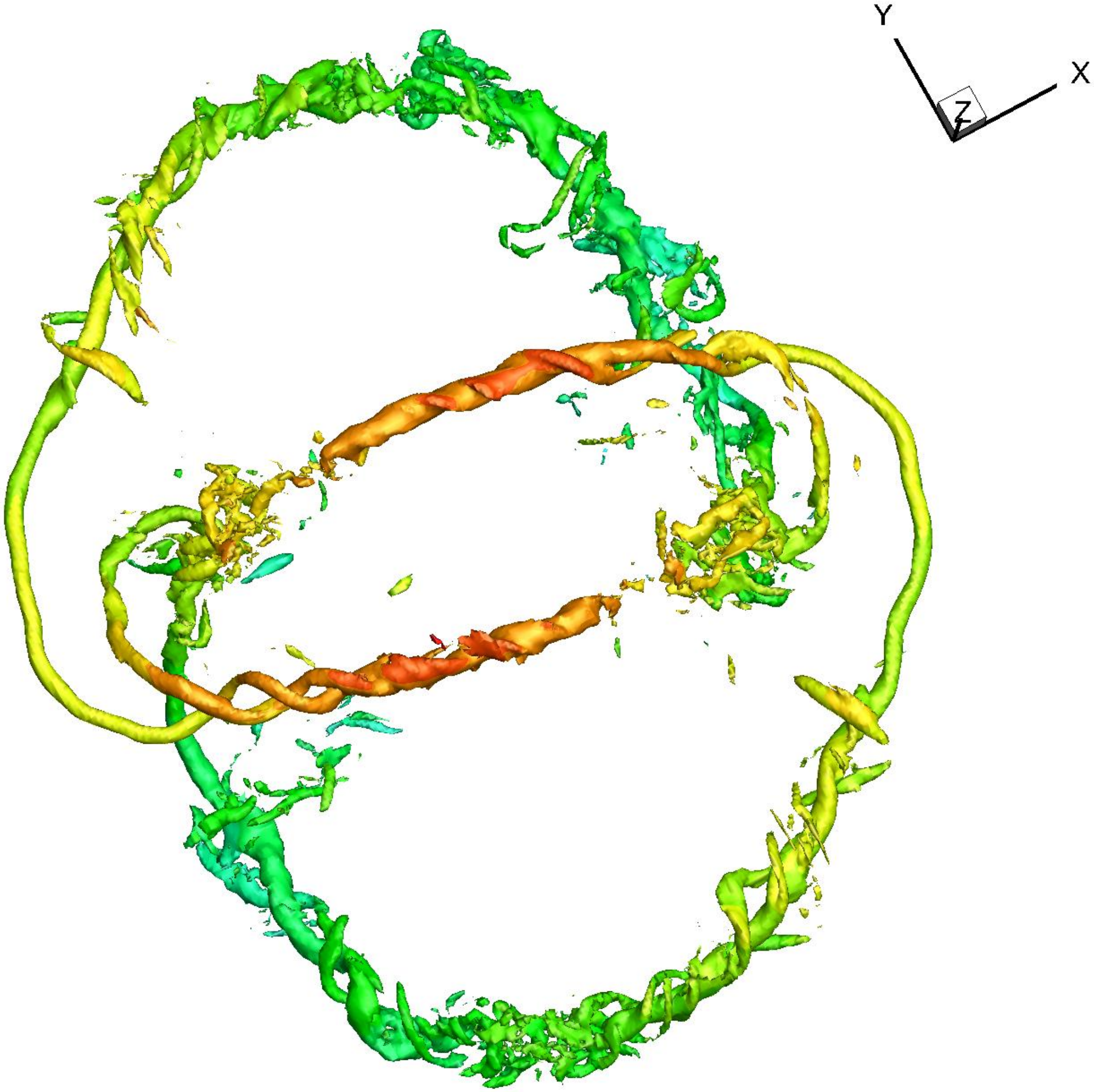
t = 61 ms


DB: vorticity_0105.h5
DB: vorticity_0105.h5
DB: vorticity_0105.h5
DB: vorticity_0105.h5

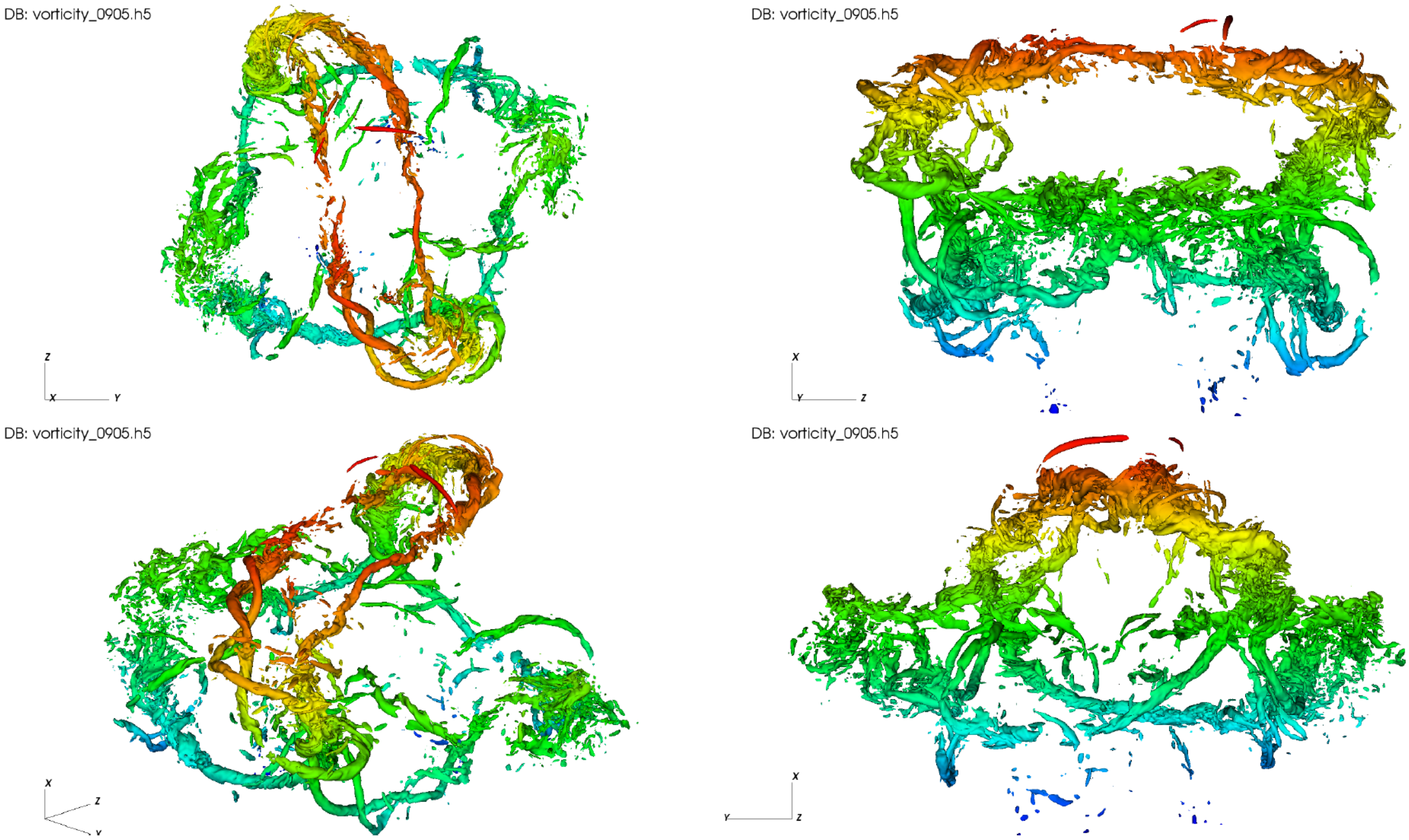
t = 294 ms


DB: vorticity_0505.h5
Z
X
Y
DB: vorticity_0505.h5
X
Y
Z
DB: vorticity_0505.h5
X
Z
Y
DB: vorticity_0505.h5
X
Y
Z

Splintering

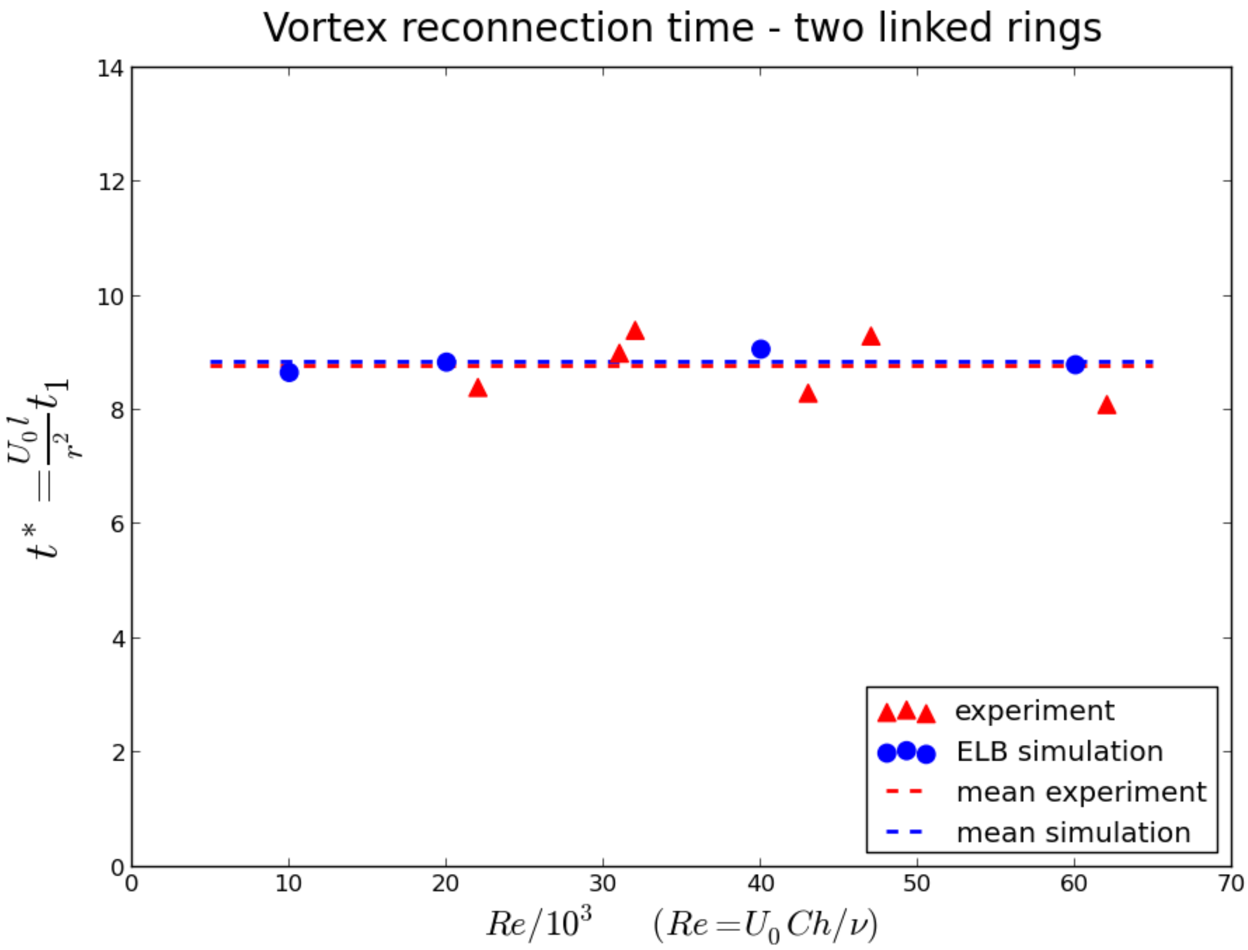


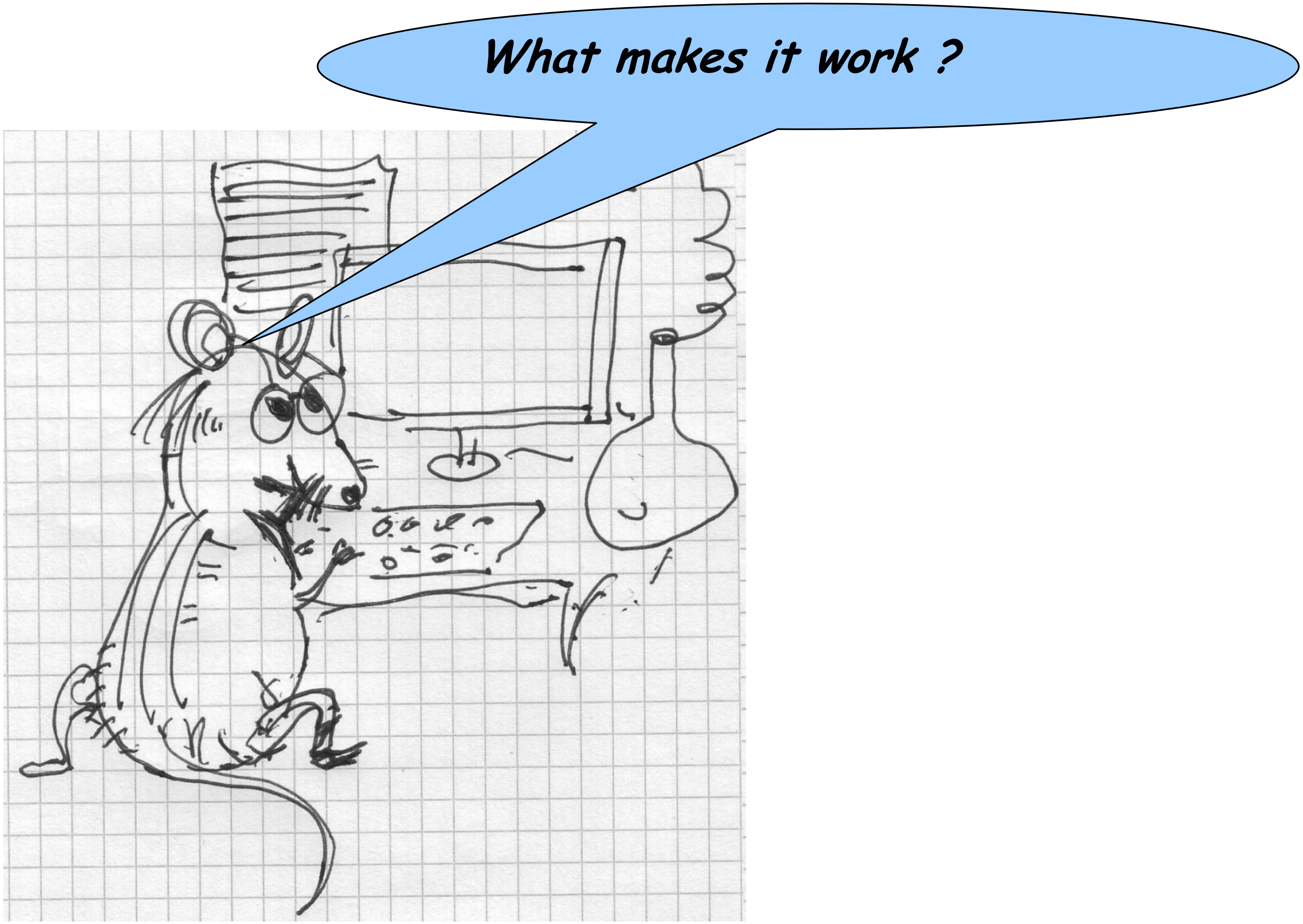
t = 526 ms



DB: vorticity_0905.h5
DB: vorticity_0905.h5
DB: vorticity_0905.h5
DB: vorticity_0905.h5

Reconnection time

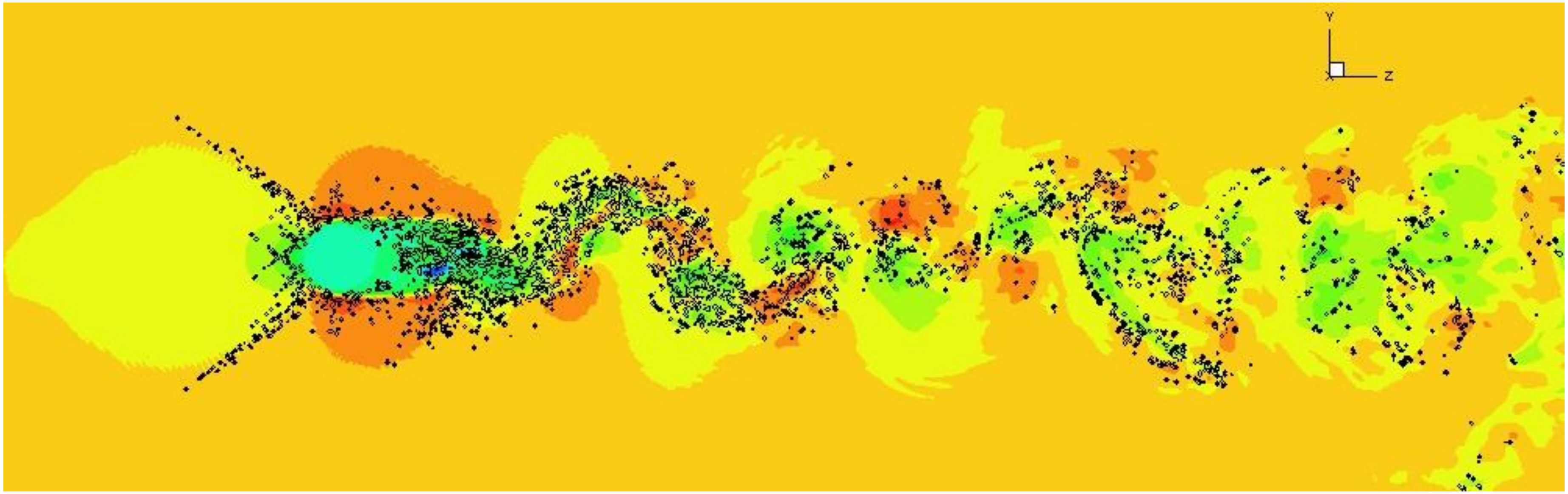



What makes it work ?

Behind the Scene: What makes ELBM work?

Flow past a circular cylinder, *Re=3300*

$$H(f + \alpha(f^{\mathrm{eq}} - f)) = H(f)$$



Color: Snapshot of stream-wise velocity
Black: Iso-contours of deviation $\alpha - 2$

Entropy Balance

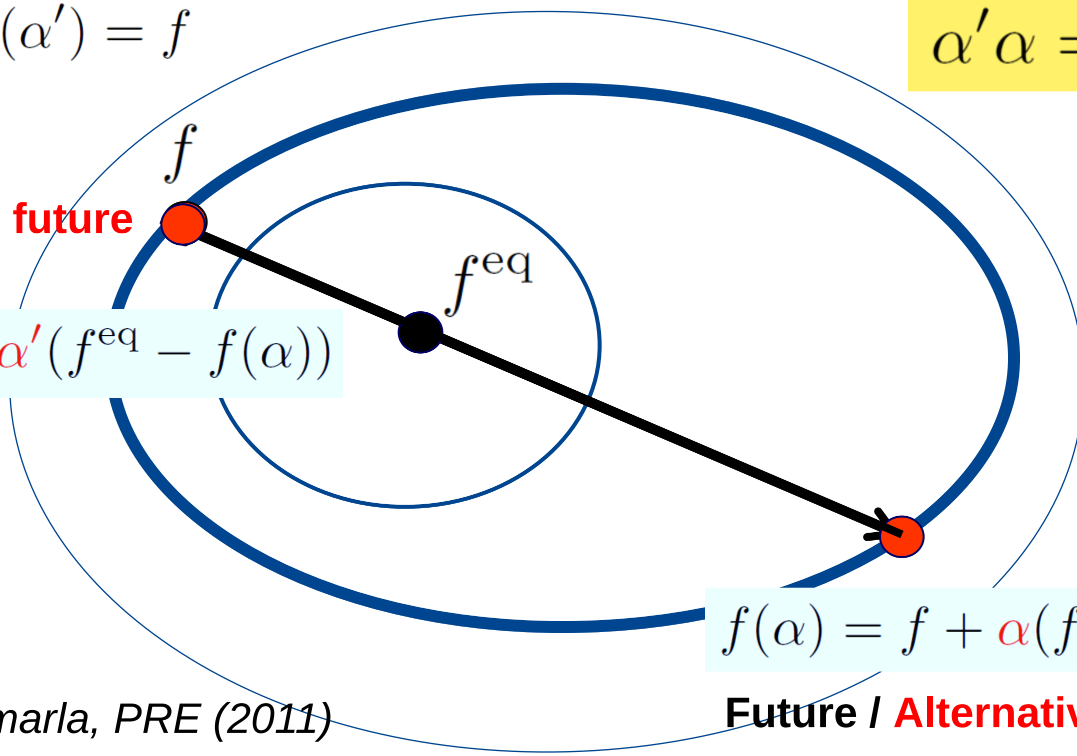
$$H(f + \alpha(f^{\mathrm{eq}} - f)) = H(f)$$

- Non-trivial solution (if exists) is always greater than 1 (over-relaxation, not equilibration)
- Over-relaxation symmetry
- Near-equilibrium (resolved) solution $\alpha = 2$ is the balance point of this symmetry

Entropic involution — **Over-relaxation symmetry**

$$f \longrightarrow f(\alpha) \longrightarrow f(\alpha') = f$$

$$\alpha'\alpha = \alpha' + \alpha$$

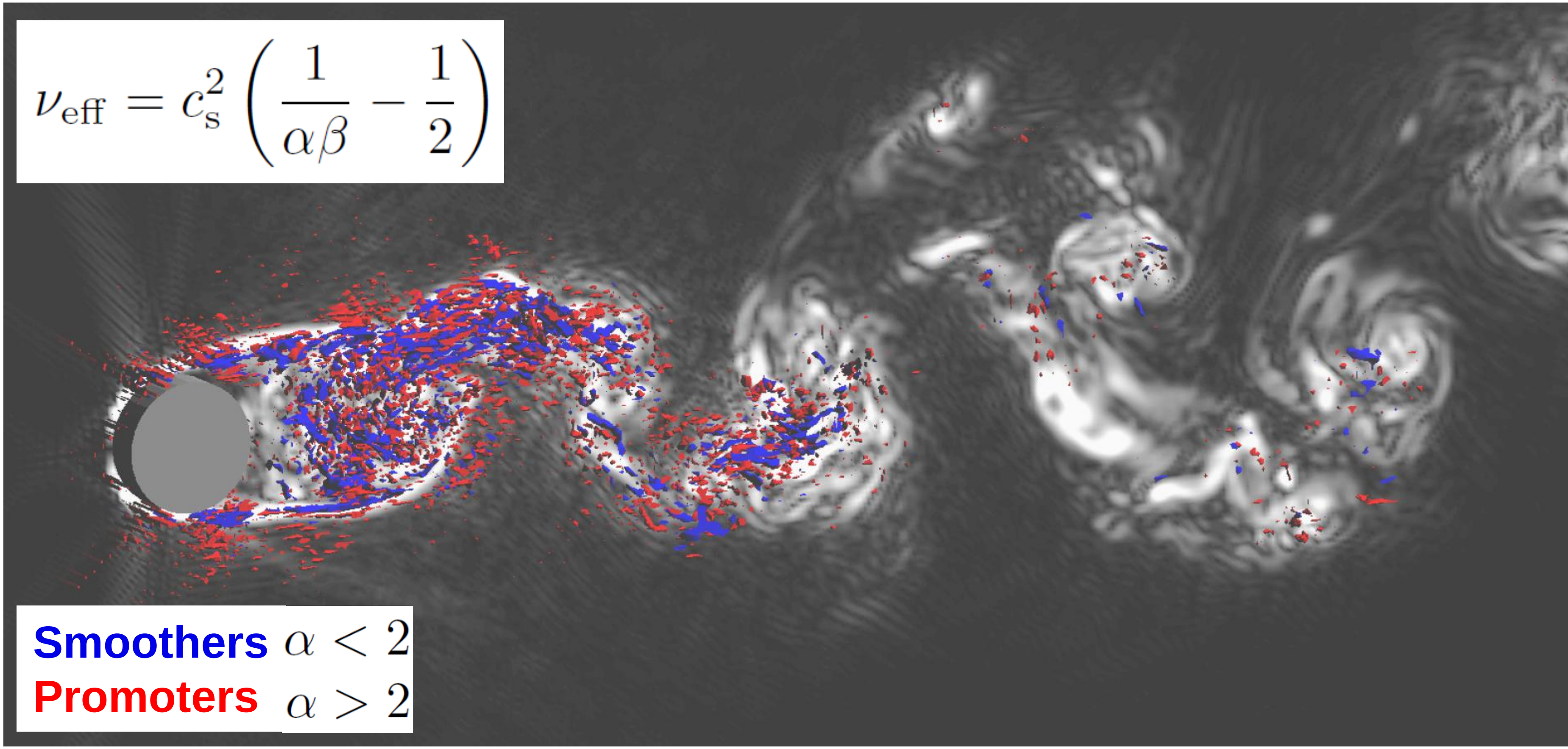


$$f(\alpha') = f(\alpha) + \alpha'(f^{\mathrm{eq}} - f(\alpha))$$

$$f(\alpha) = f + \alpha(f^{\mathrm{eq}} - f)$$

Karlin, Succi, Chikatamarla, PRE (2011)

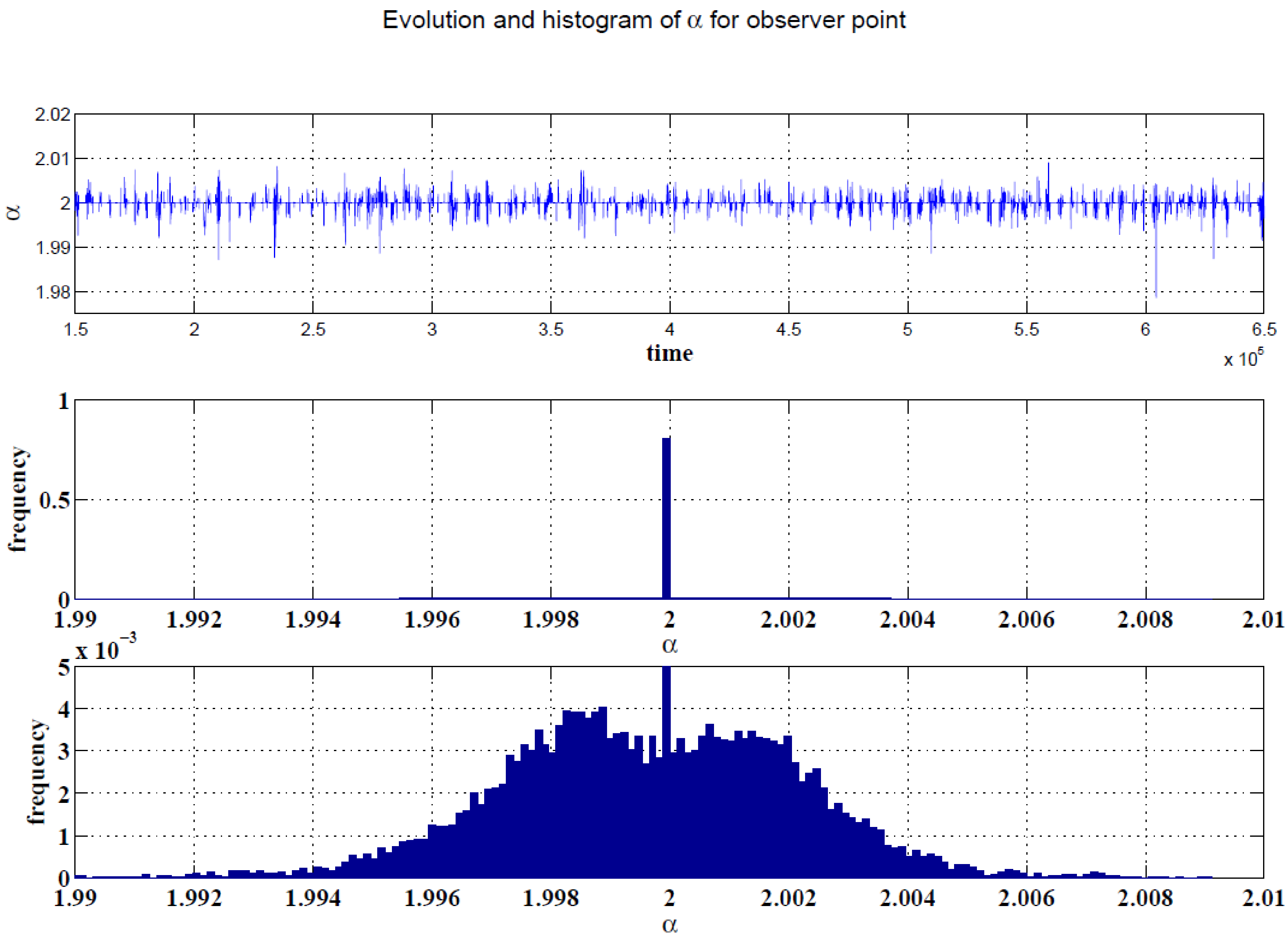
The mechanism of ELBM:
Pepper viscosity



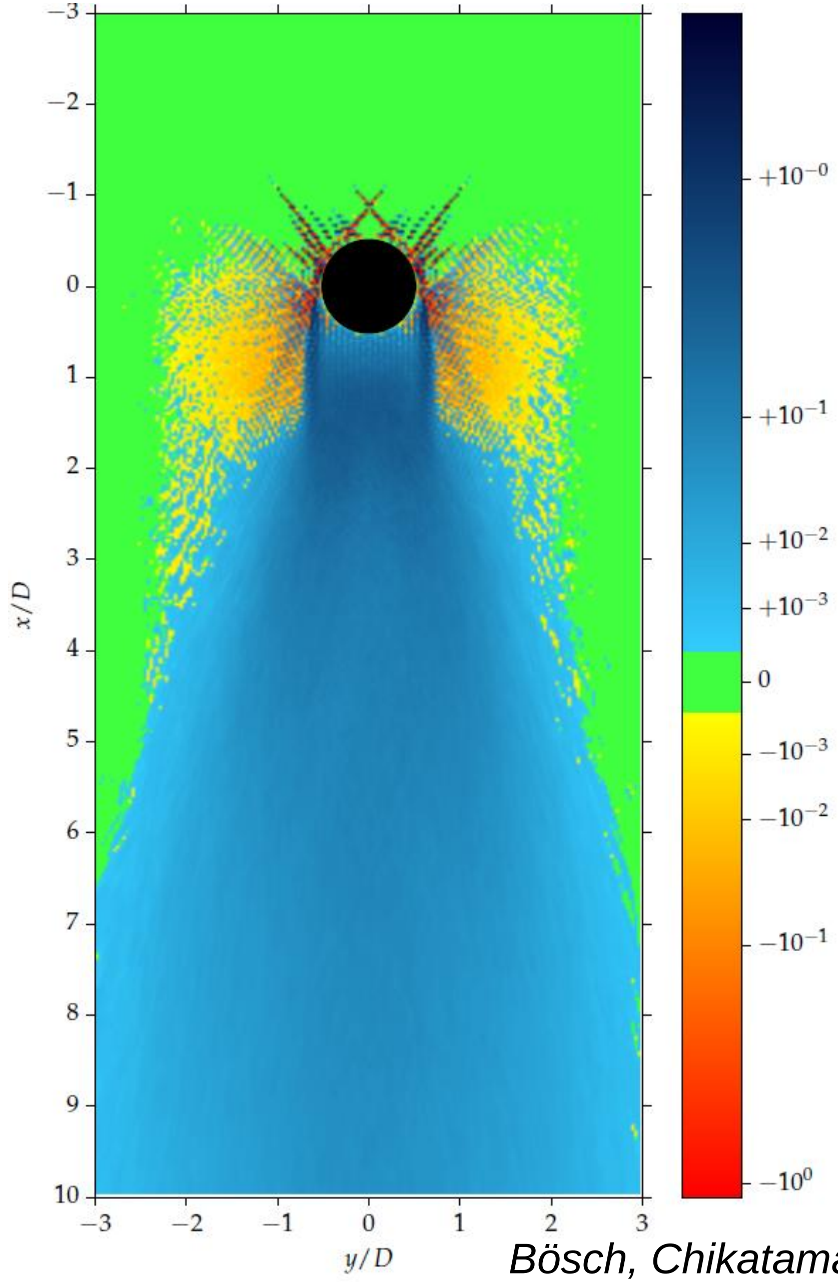
Promoters (**sharpening** of the velocity gradient at some nodes through **decreasing** the **effective viscosity**) is the implication of the entropy condition and is the “trade secret” of the Entropic LB for maintaining the **accuracy** while the **smoothers** are taking care of the **stability** through an **increase** of the **effective viscosity**

Bösch, Chikatamarla, Karlin, Succi (2014)

Over-relaxation history at a typical lattice node



Guardian Angel



Time-averaged effective viscosity

$$R = \frac{\nu_{\text{eff}} - \nu}{\nu}$$

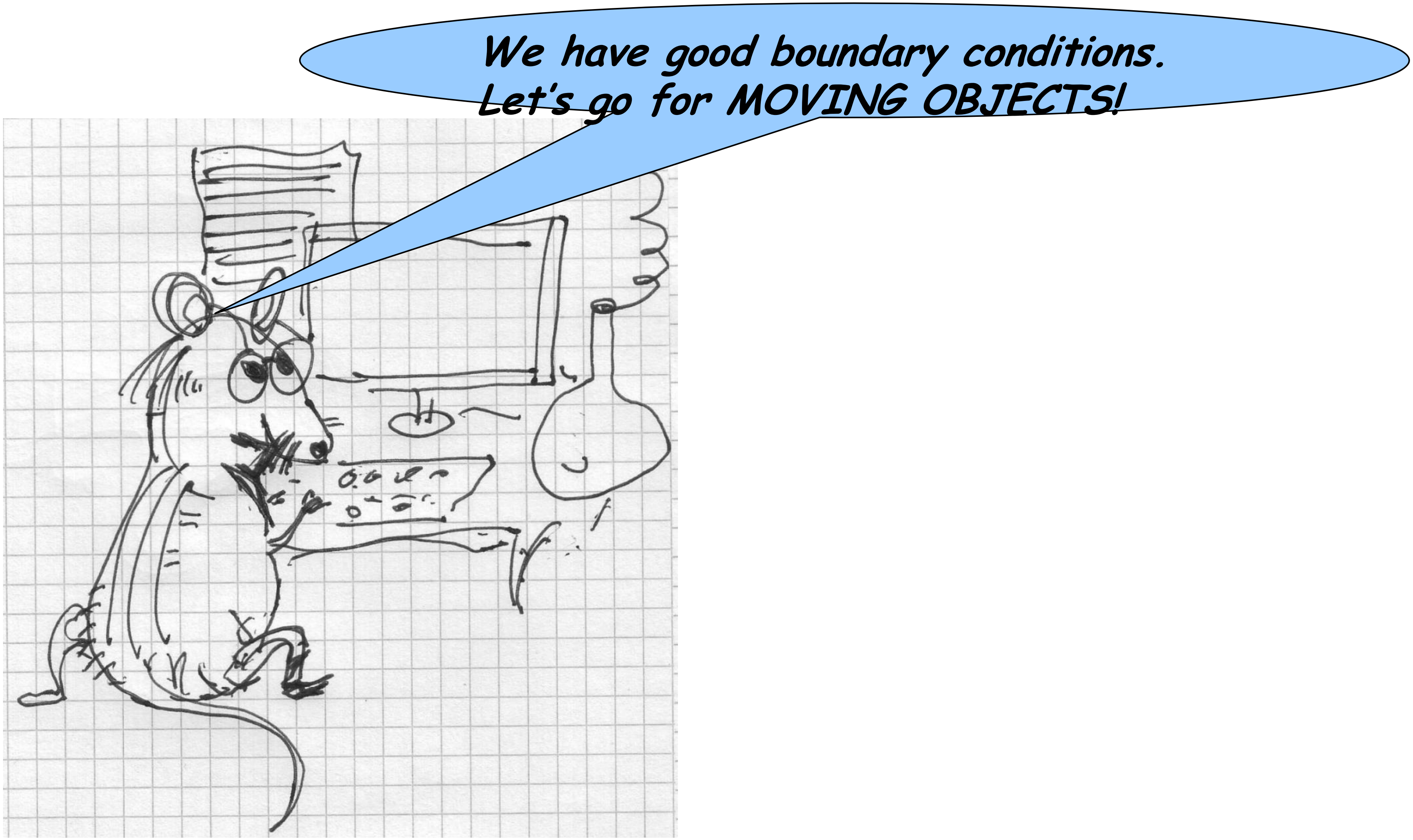
Bösch, Chikatamarla, Karlin, Succi (2014)

Questions re. **entropic limiter LBM** [Gorban, Brownlee, Levesley, Packwood]

Q: Is ELBM a special case of a limiter?
A: No.
Fluctuating effective viscosity (by H-balance) instead of adding viscosity.
Prevention (ELBM) instead of **rescuing** (limiter LBM).

Q: What to do if entropy condition has no solution?
A: Terminate your simulation.
In ELBM simulations of fluid dynamics this never occurres.

Q: Is a “better” H-function possible?
A: No.
There is **only one** convex function H, the minimizer of which under fixed density and momentum **implies** the correct pressure tensor.
[Karlin, Ferrante, Oettinger, Europhys. Lett. 1999]



Dorschner, Bösch, Chikatamarla, Karlin, (2014)

Stationary Cylinder Validation

- **Drag and Lift Coefficient:**

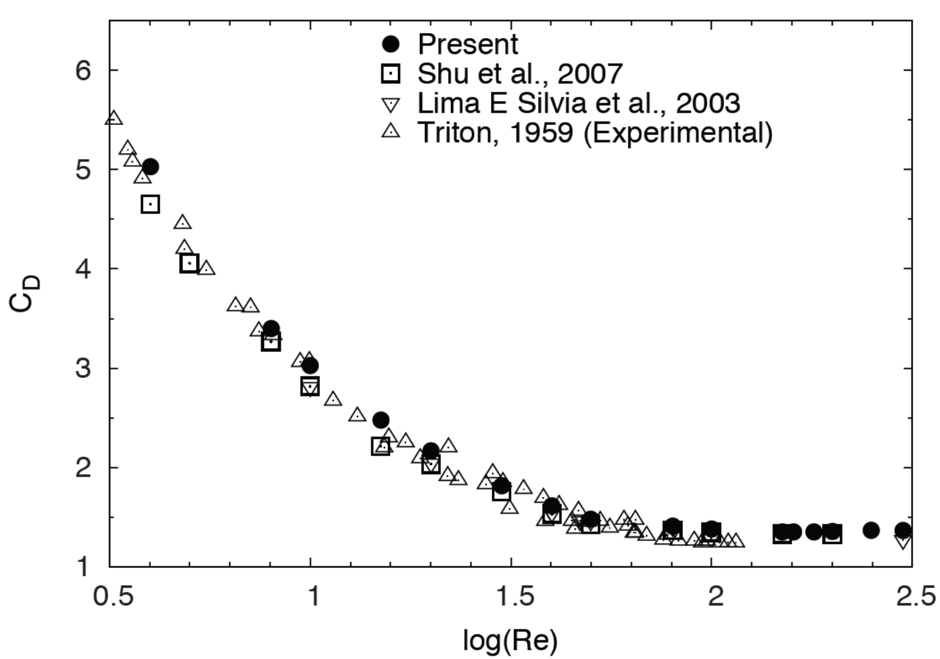
$$C_D = \frac{F_D}{\frac{1}{2}\rho_\infty u_\infty^2 D}, \qquad C_L = \frac{F_L}{\frac{1}{2}\rho_\infty u_\infty^2 D},$$

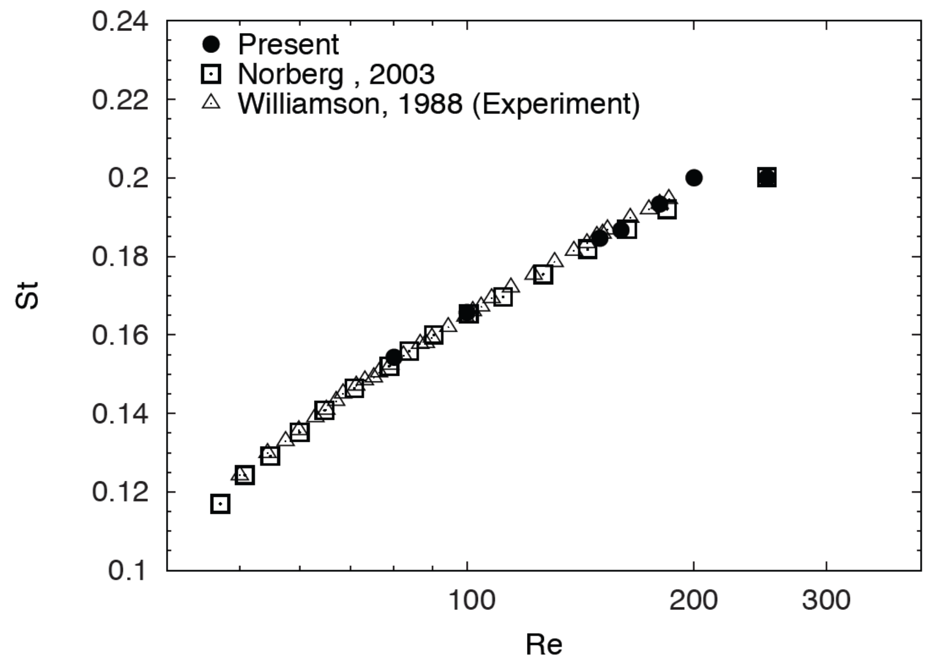
where F_D and F_L are the forces in stream-wise and transverse direction.

- **Strouhal number:**

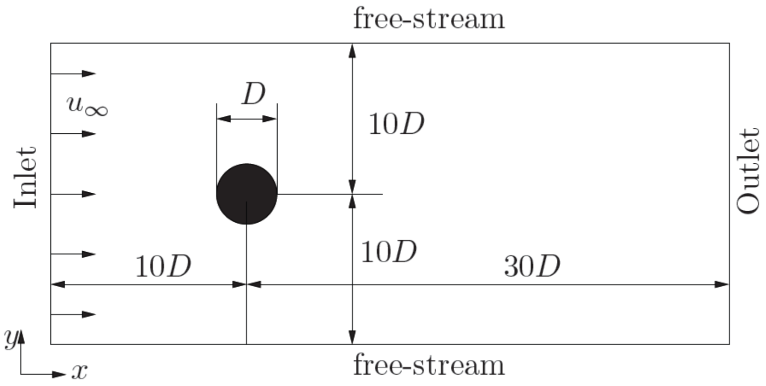
$$St = \frac{f_s D}{u_\infty},$$

where f_s is the vortex-shedding frequency.





Transversly Oscillating Cylinder



- **Prescribed motion:**

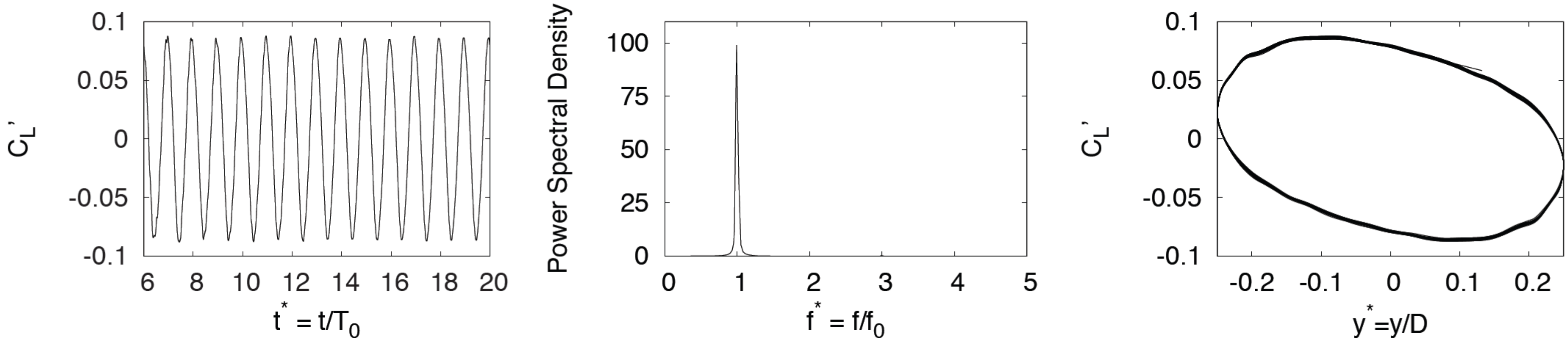
$$y(t) = y_0 + y_{max}\sin(2\pi f_0 t),$$

with

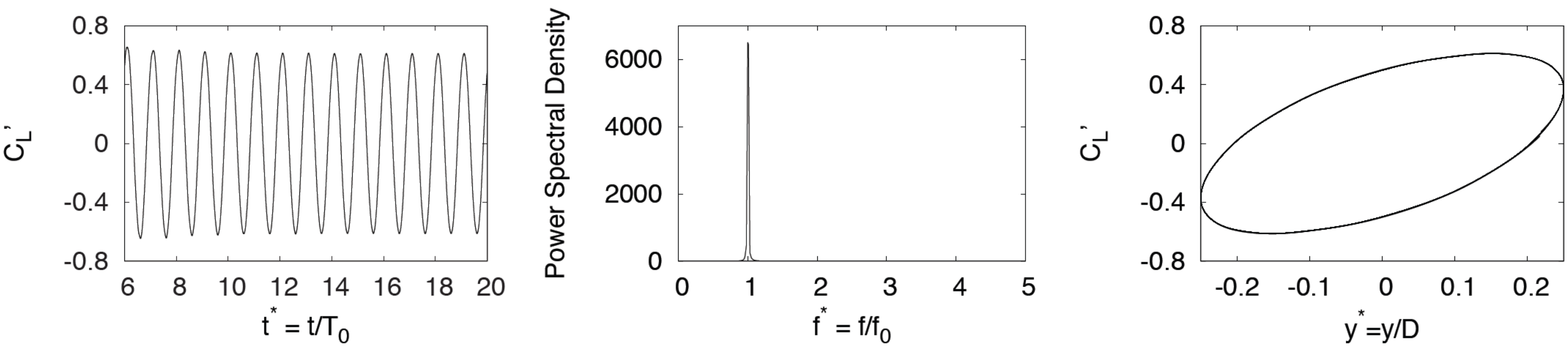
- $A = y_{max}/D = 0.25$, $F = f_0/f_s$

Video withheld due to size

Transversly Oscillating Cylinder: Lock-in Regime

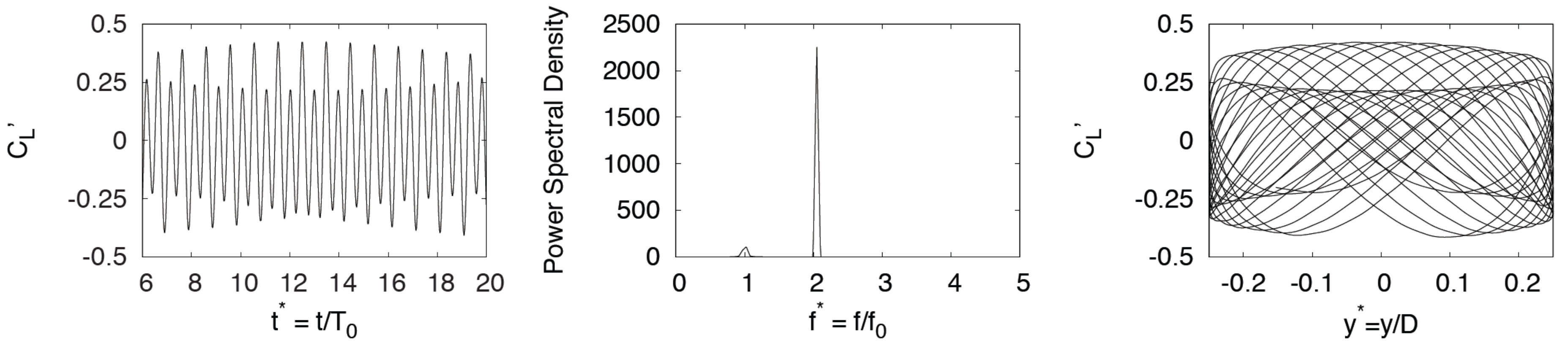


(a) $F = 0.9$

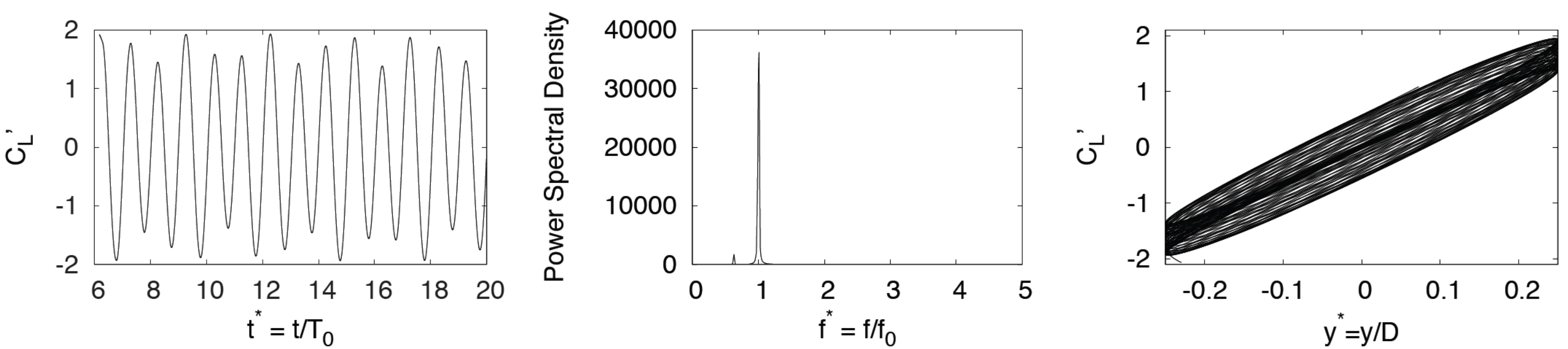


(b) $F = 1.1$

Transversly Oscillating Cylinder: Unlocked Regime



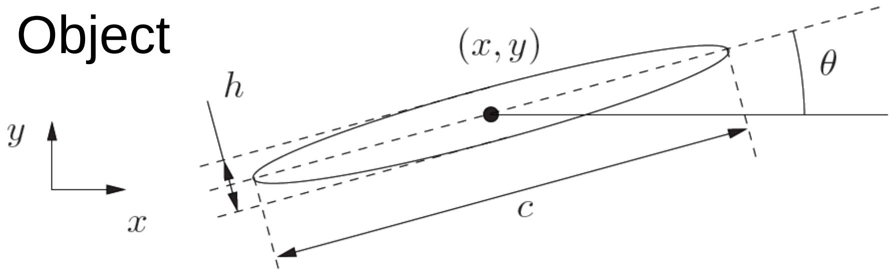
(c) $F = 0.5$



(d) $F = 1.5$

Flapping Wing

Object

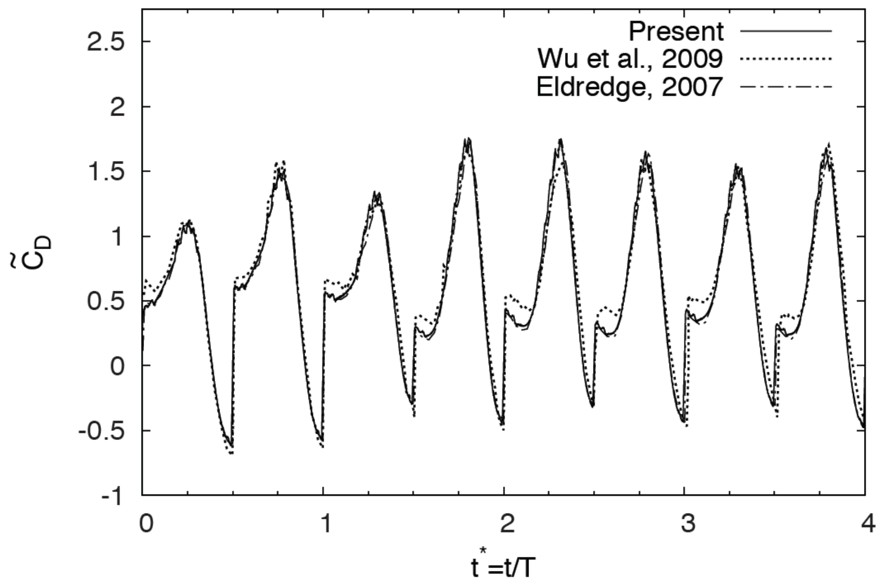


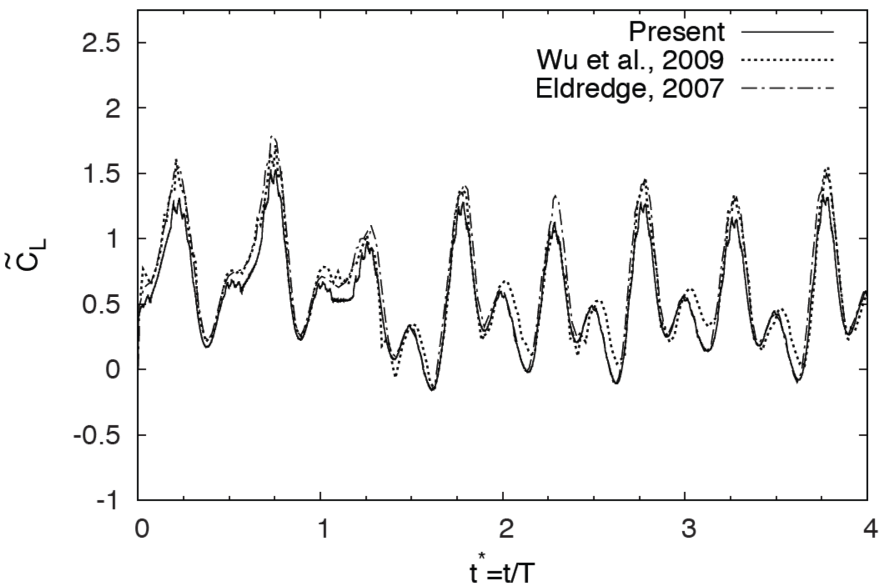
Prescribed motion

$$x(t) = \frac{A_0}{2}\cos(2\pi f t),$$
$$\theta(t) = \theta_0 + \beta \sin(2\pi f t + \phi),$$

Video withheld due to size

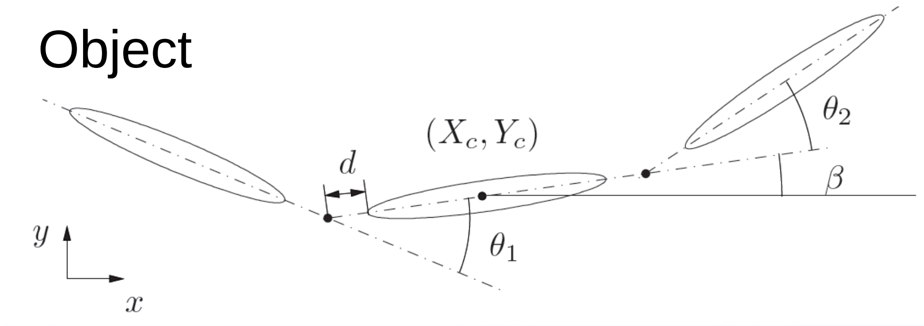
Lift and Drag Coefficients





Three - Linkage Fish

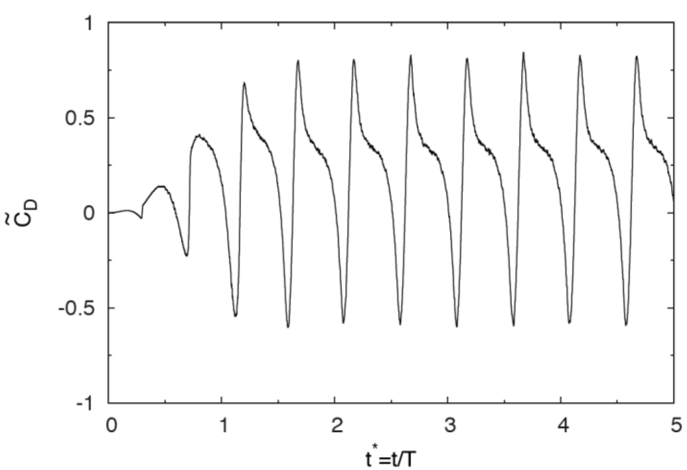
Object

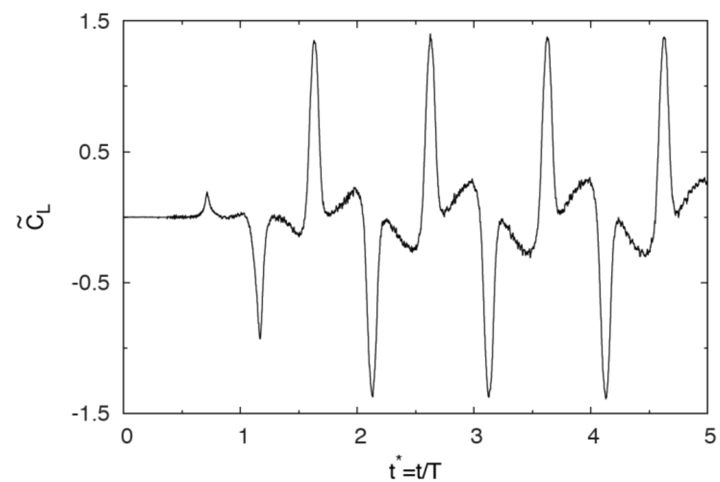


Prescribed motion

$$\theta_1(t) = \sin(2\pi f t),$$
$$\theta_2(t) = \cos(2\pi f t),$$

Video withheld due to size

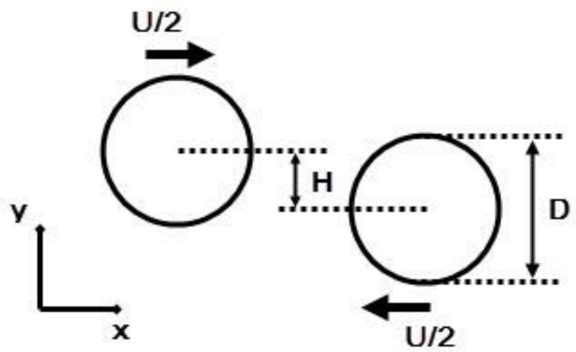




Can ELBM help with multiphase flows?

Mazloomi, Chikatamarla, Karlin, (2014)

Droplets Collision



Weber number

$$\mathrm{We} = \frac{\rho_l D U^2}{\sigma}$$

Reynolds number

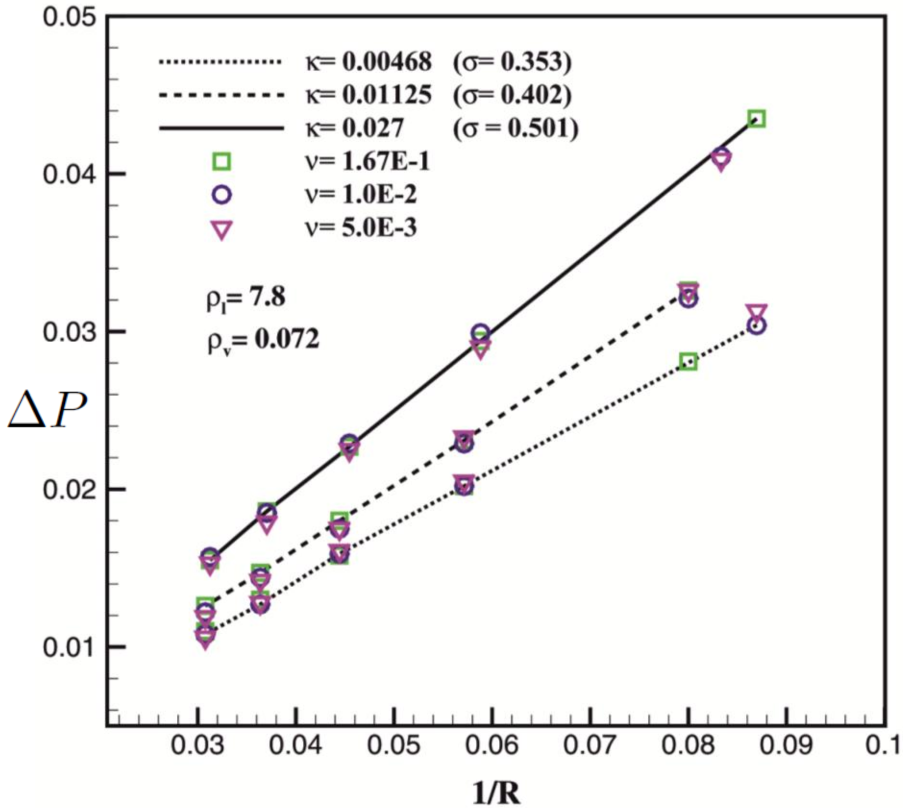
$$\mathrm{Re} = \frac{\rho_l D U}{\mu}$$

Impact parameter

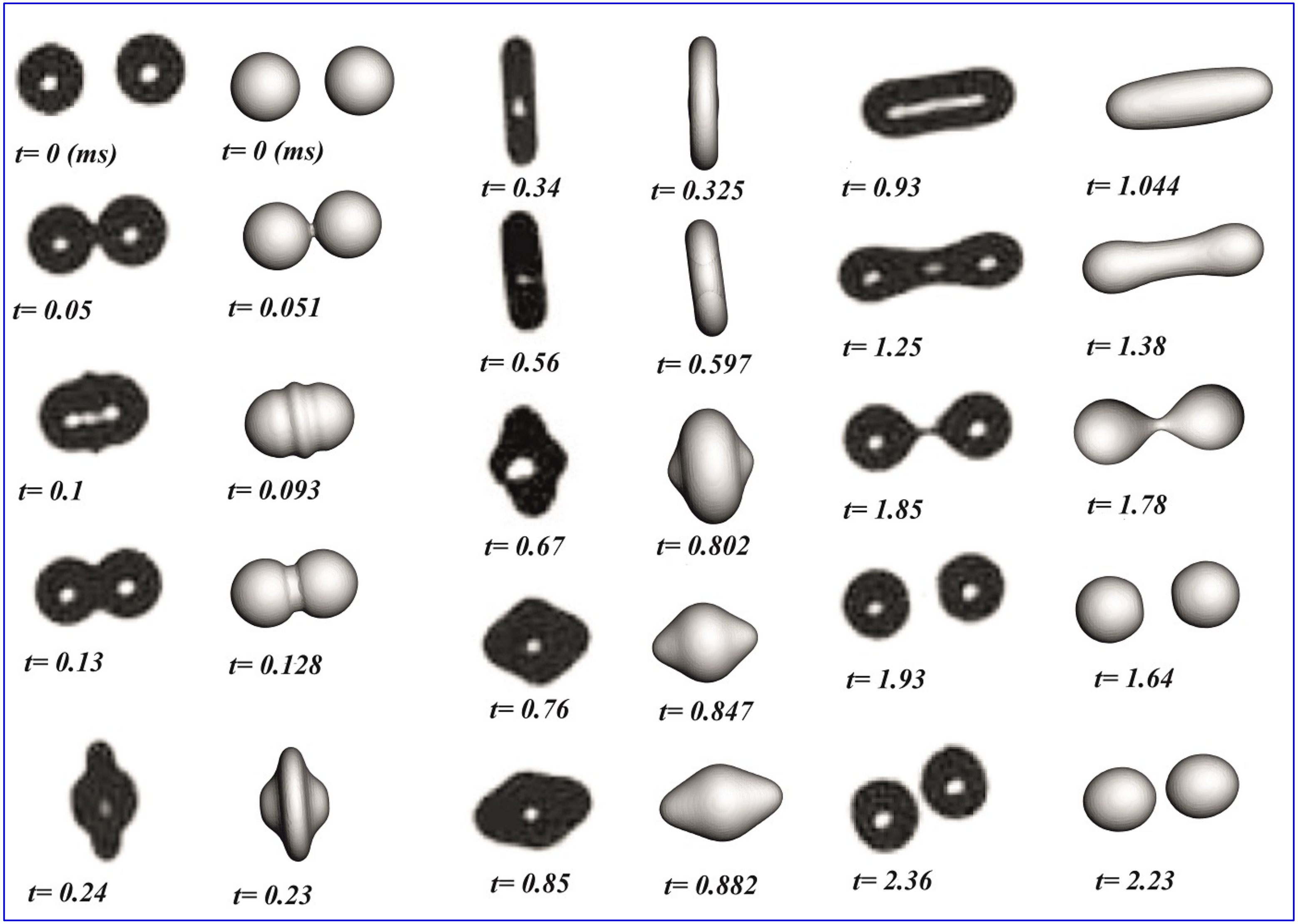
$$B = \frac{H}{D}$$

Validation: Laplace's Law

$$\Delta P = \frac{\sigma}{R}$$



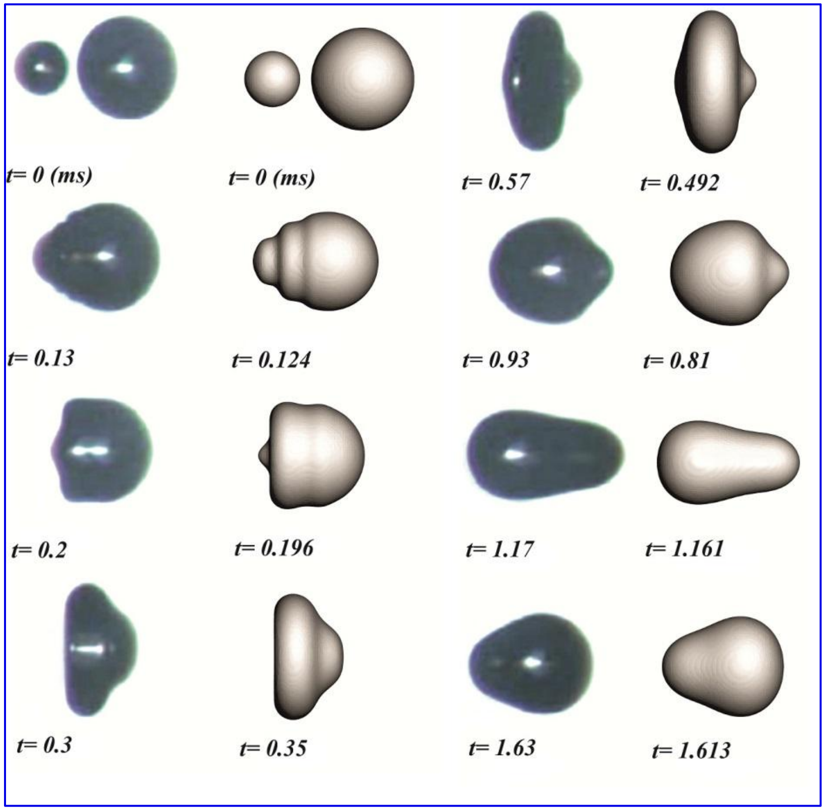
$$\mathrm{We} = 37.2,\ \mathrm{Re} = 228,\ B = 0.01,\ \rho_{\mathrm{l}} = 7.85,\ \rho_{\mathrm{v}} = 0.063,\ \nu = 1.7 \times 10^{-2}$$



Qian, J., and C. K. Law. "**Regimes of coalescence and separation in droplet collision**." *J. Fluid Mech.* 331 (1997): 59-80.

Unequal-size Droplets Head-on Collision

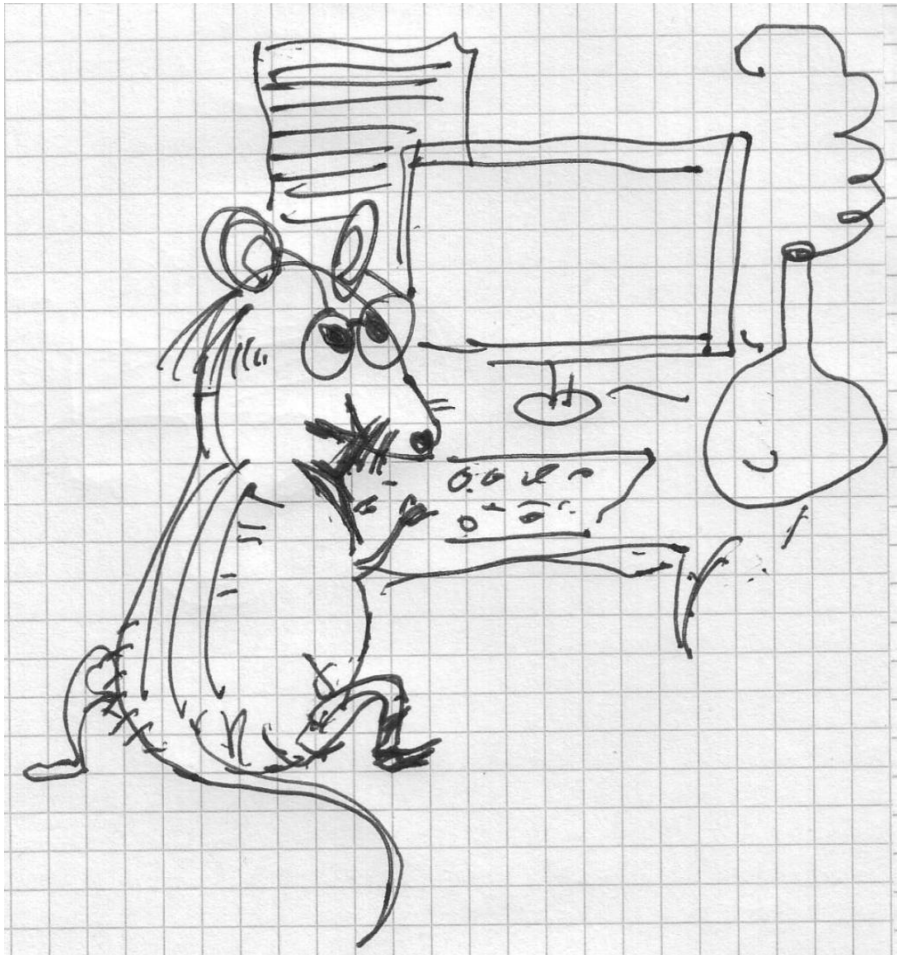
$$\mathrm{We} = 17.6,\ \mathrm{Re} = 185,\ B = 0,\ R_1/R_2 = 1.87, \rho_\mathrm{l} = 7.85,\ \rho_\mathrm{v} = 0.063,\ \nu = 1.83 \times 10^{-2}$$

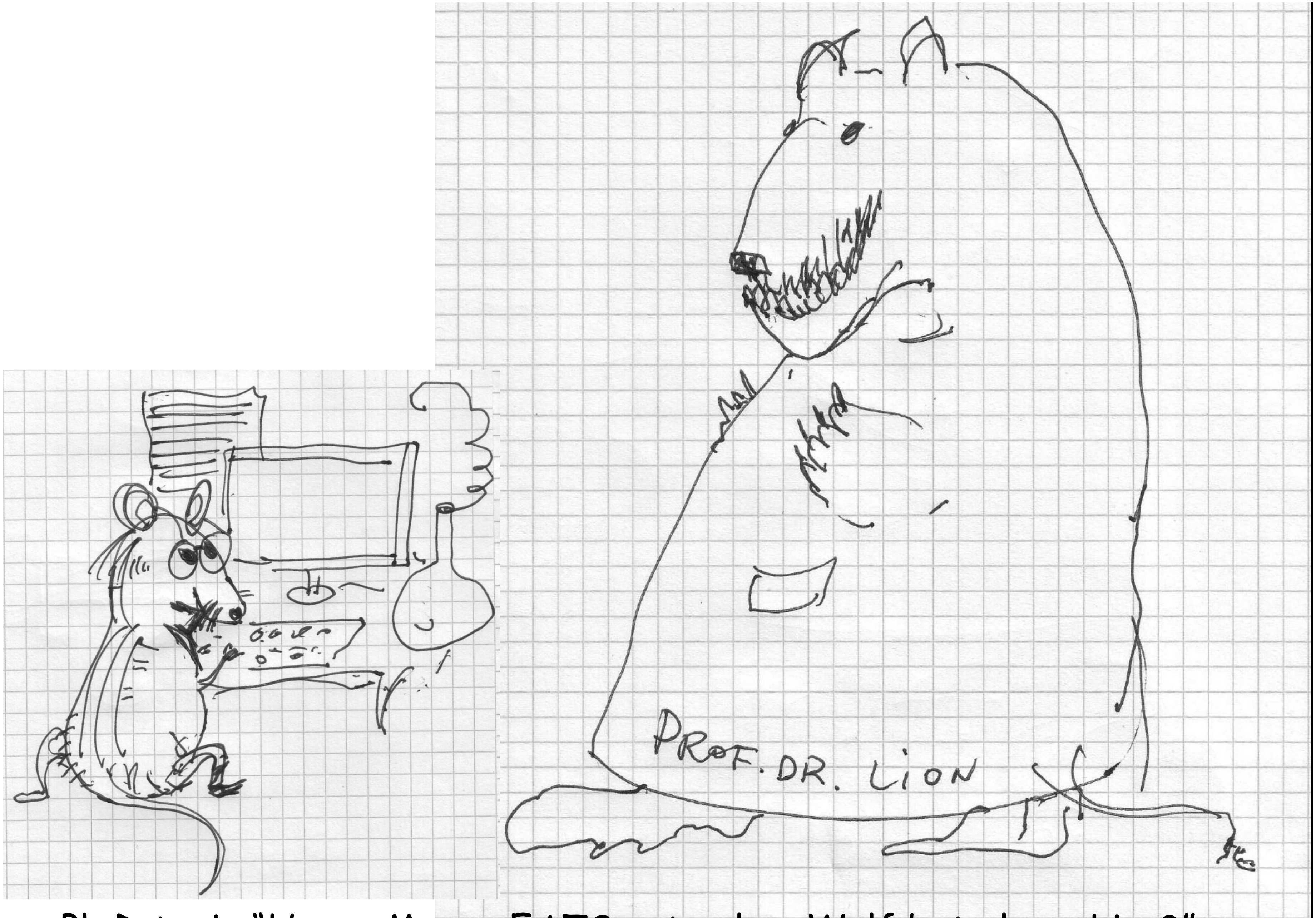


Tang, Chenglong, Peng Zhang, and Chung K. Law. **Bouncing, coalescence, and separation in head-on collision of unequal-size droplets**, *Physics of Fluids* 24.2 (2012): 022101.

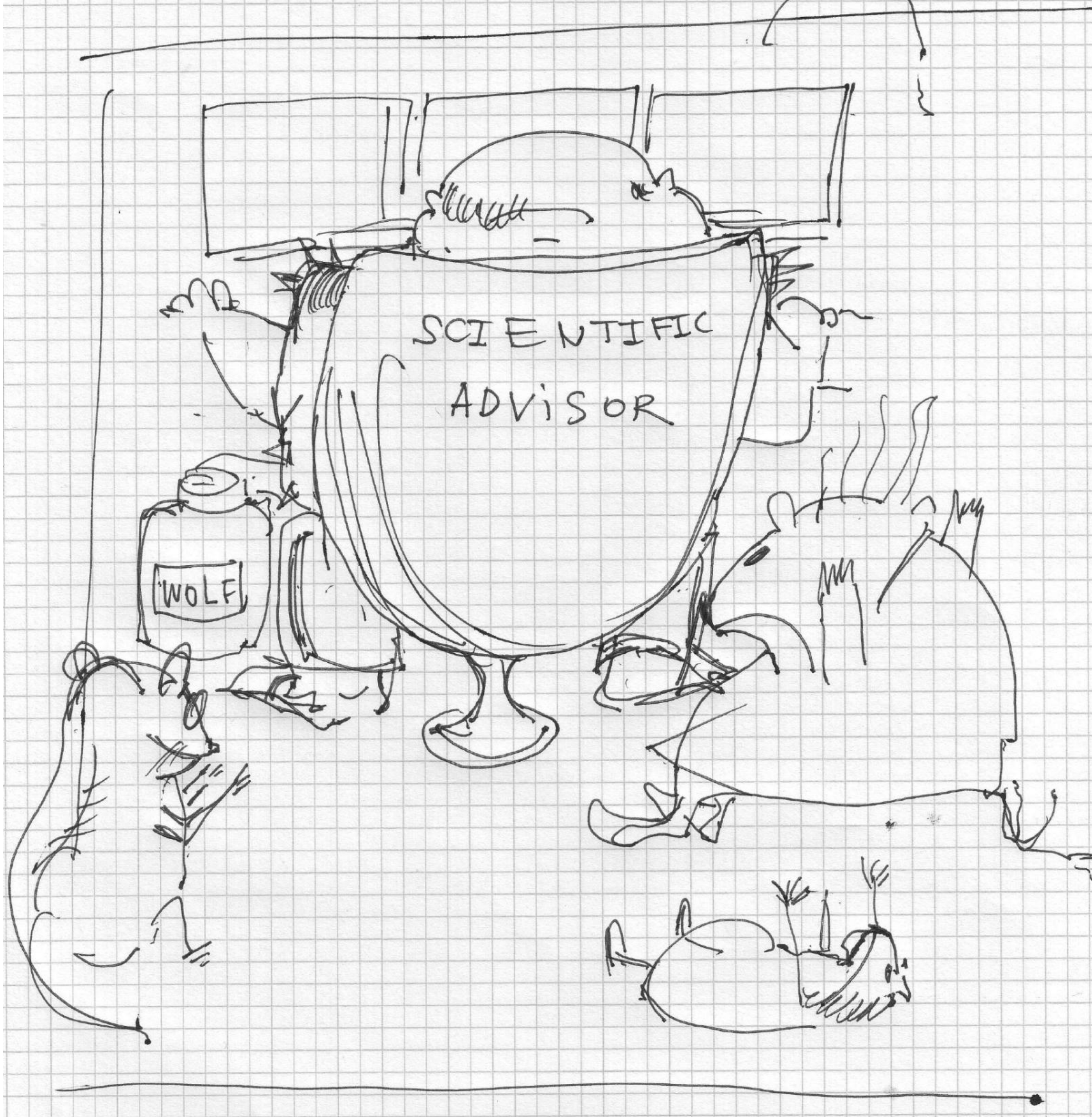

Can we do it better ?

Ph D topic "How a Mouse EATS a Wolf?"





Ph D topic “How a Mouse EATS not only a Wolf but also a Lion?”


SCIENTIFIC
ADVISOR
WOLF

Conclusions:

1. It does not matter WHAT the topic of your Ph D thesis is.
2. It does matter WHO your Scientific Advisor is.

With Scientific Advisor, Krasnoyarsk 1992