Extreme model reduction in neuroscience: Principles before Realism

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Overview

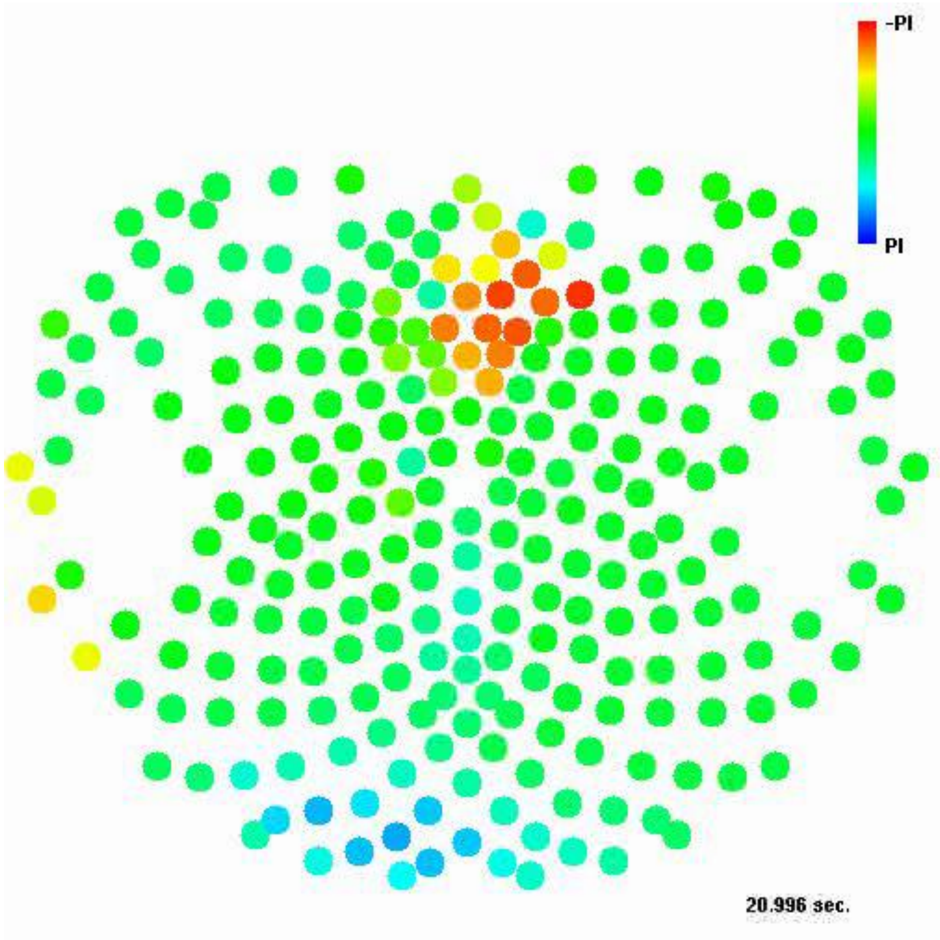
- Function: patterns of brain activity
- Structure: patterns of connectivity
- Structure-function relationship: a *Proof of concept* model
- Efforts to enhance model realism
- Conclusions

Function

Patterns of Brain Activity

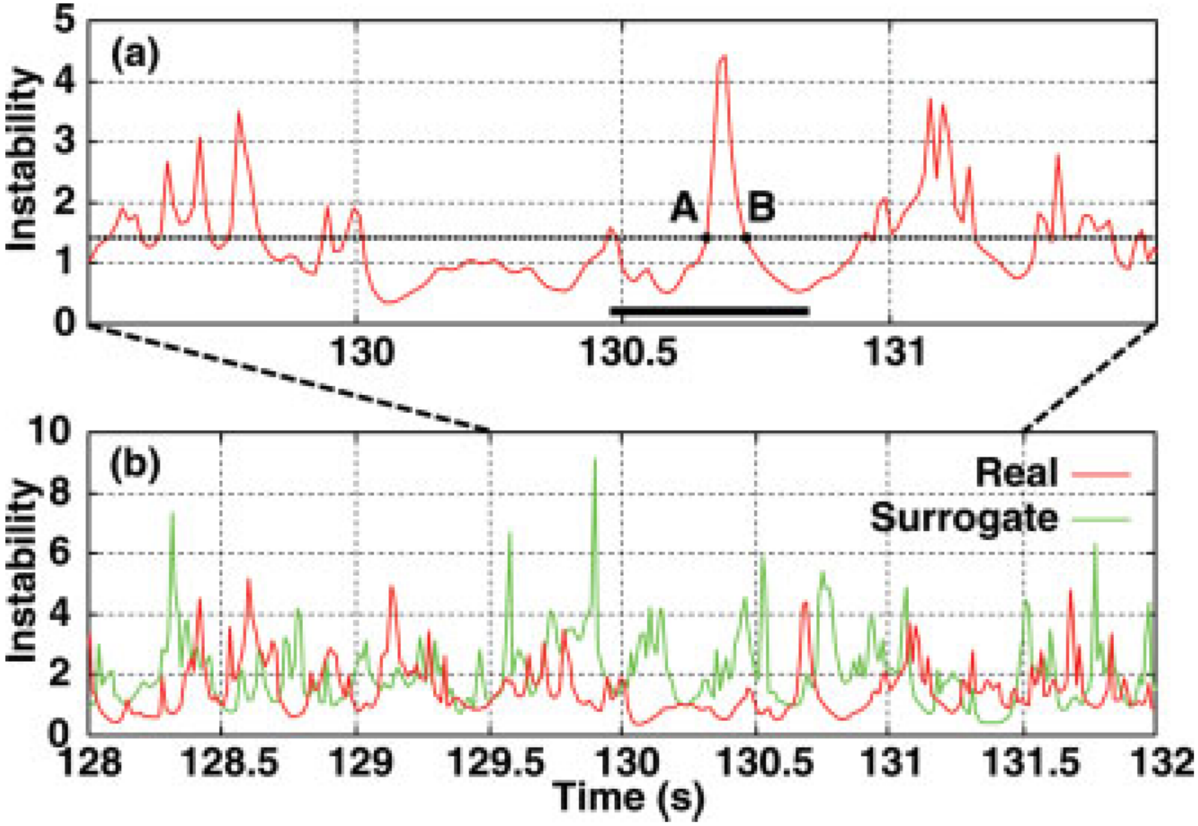
Spontaneous Phase Patterns

Ito et al. 2007



Traveling and Standing waves

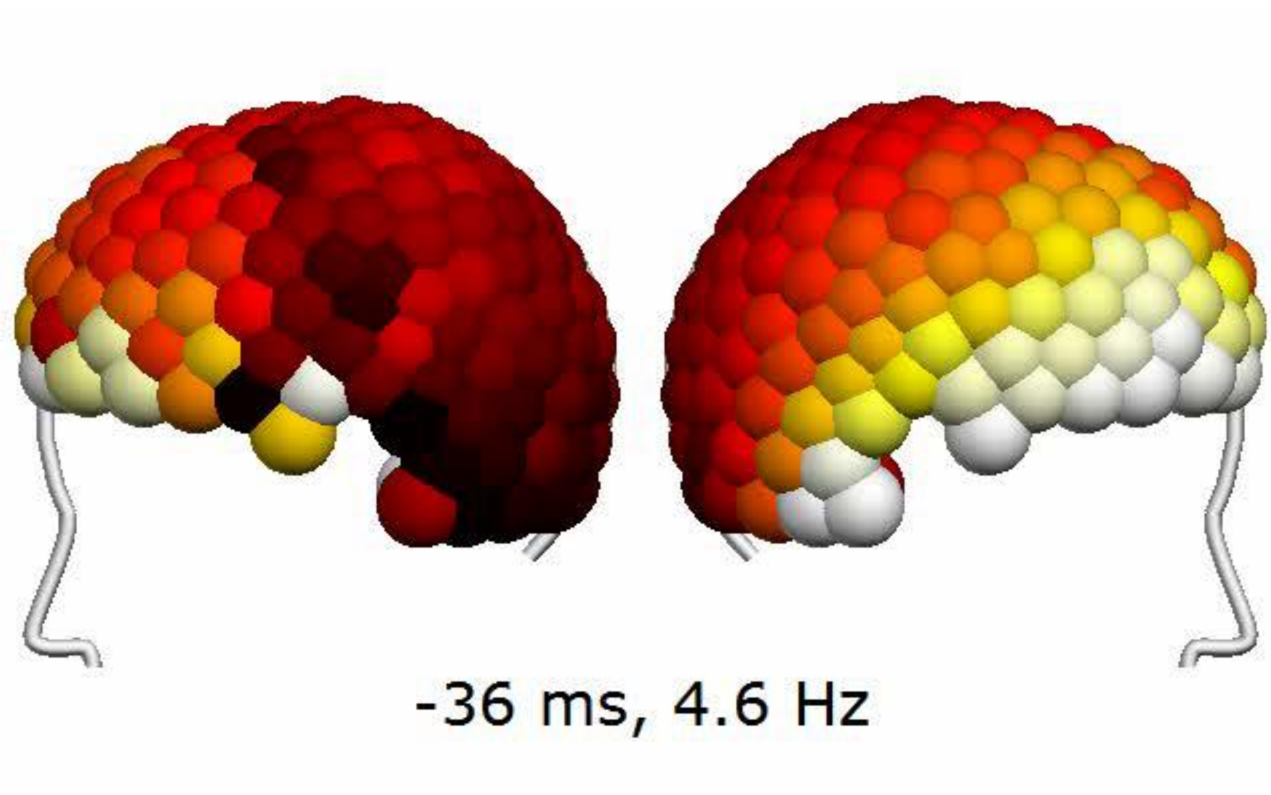
Ito et al. 2007



Instability index of relative phase

Evoked Phase Patterns

Alexander et al. 2013.



Brain Function

- Phase patterns alternate between local and global modes
- Spontaneous and evoked

Structure

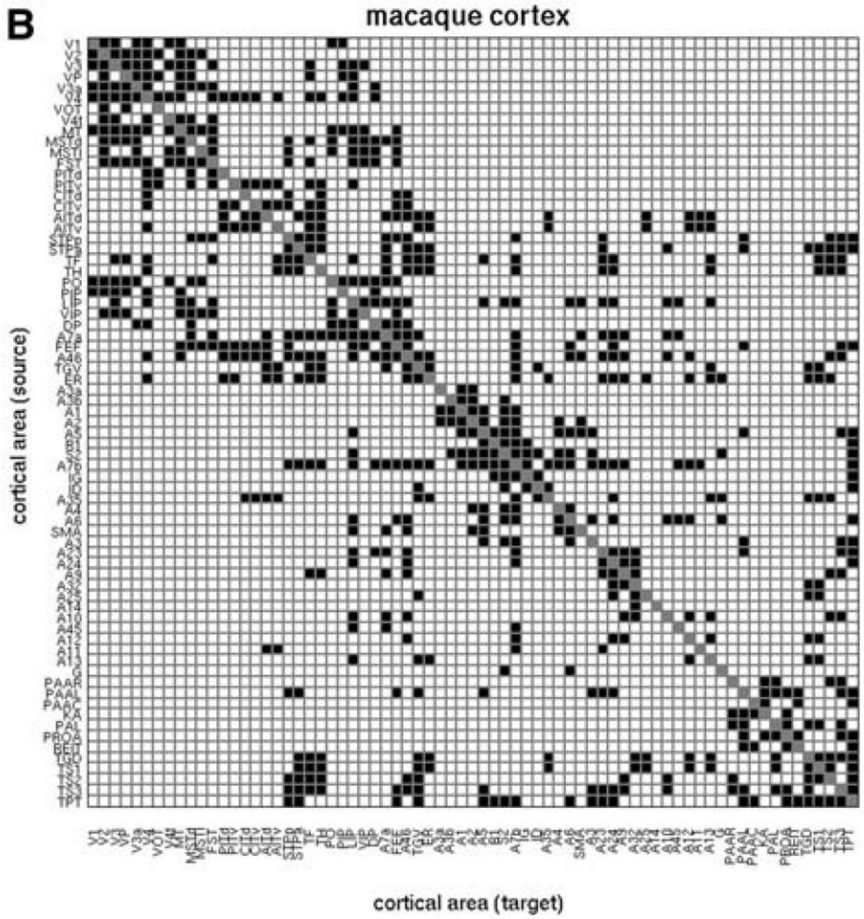
Patterns of Brain Connectivity

Connectivity

Macaque Cortex

(Young, 1993; Sporns & Zwi, 2004)

Topology	Path Length	Cluster Index
MC	2.3769	0.4614
Random	2.0310 (0.0051)*	0.1497(0.0030)*
Lattice	3.8262 (0.0099)*	0.6593 (0.0002)*
Rand(io)	2.1159 (0.0133)*	0.2409 (0.0047)*
Latt(io)	2.8901 (0.1173)*	0.8992 (0.0211)*



Connectivity Structure

- The brain as a Modular Small World
- Unlikely to be entirely pre-programmed
- Self-organized and dynamically maintained
- Evolving due to continued plasticity

A hypothesis about structure-function relationship

Symbiosis:

The dynamics maintains the connectivity

The connectivity supports the dynamics

→

Dynamics on Evolving Network

Proof of concept

Extreme Model Reduction

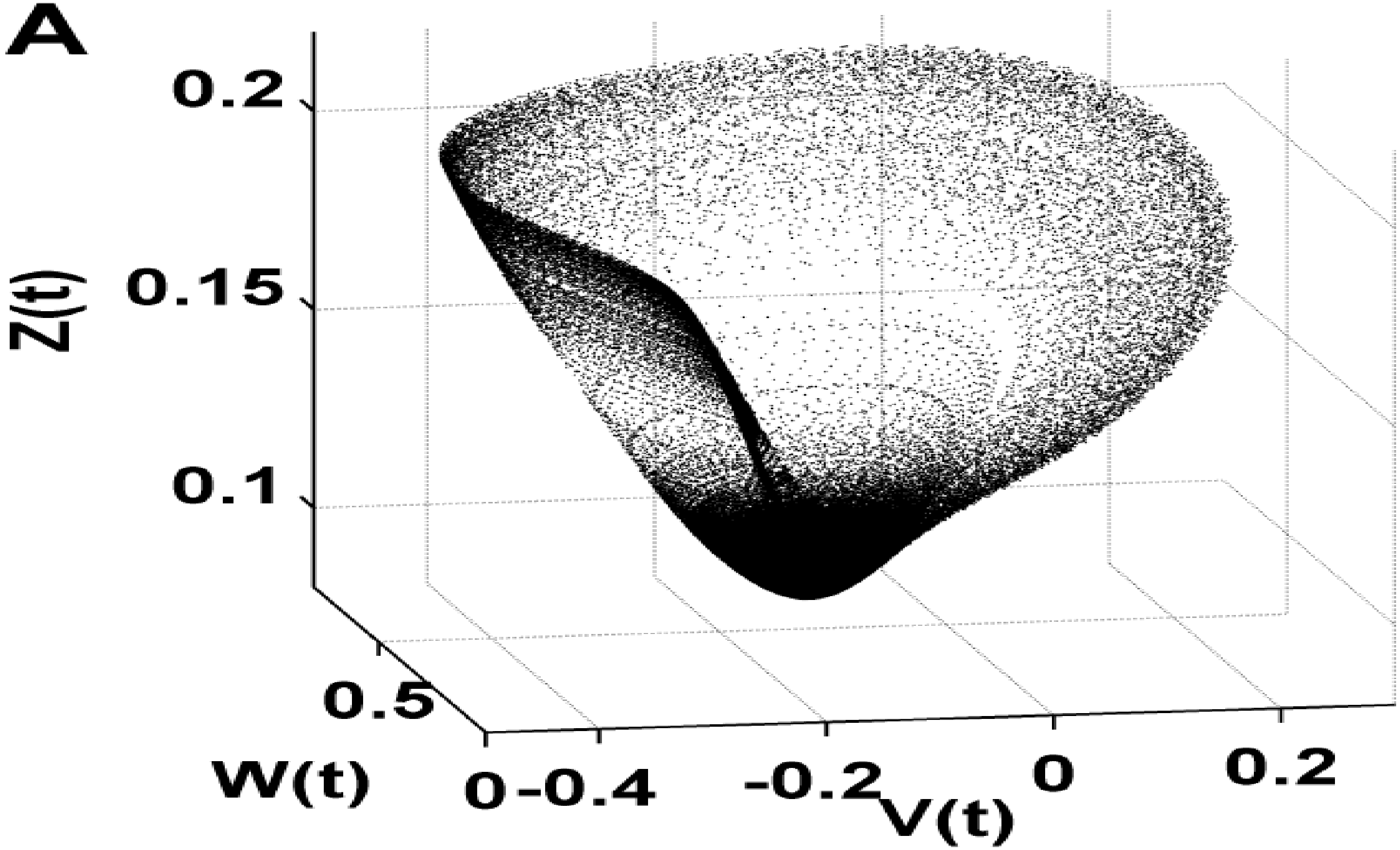
From regional activity to one-dimensional maps

From white-matter connectivity to undirected, unlabeled graphs

Gong & van Leeuwen, 2004; van den Berg & van Leeuwen, 2004; Rubinov et al., 2009).

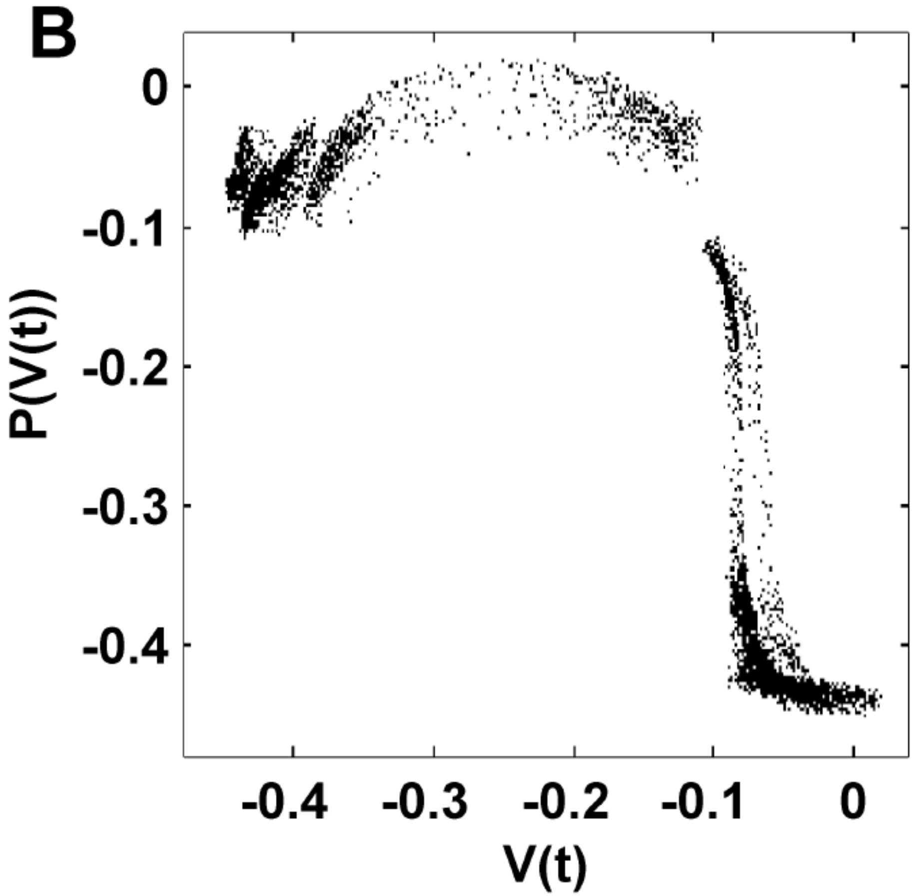
Neural Mass Model

Breakspear et al. 2003

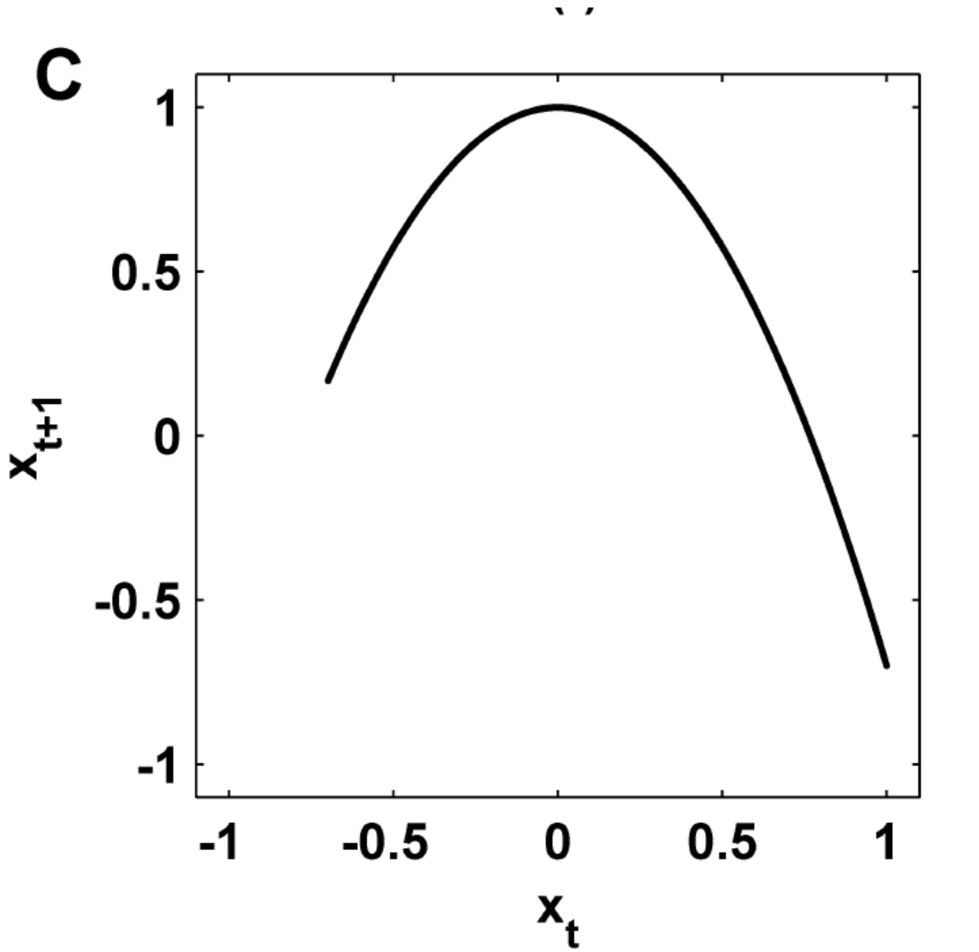


Return plot in three dimensions. Potential of pyramidal (*V*) and inhibitory (*Z*) neurons, average number of open potassium ion channels (*W*)

Poincaré section of the Mass Model



Logistic Map

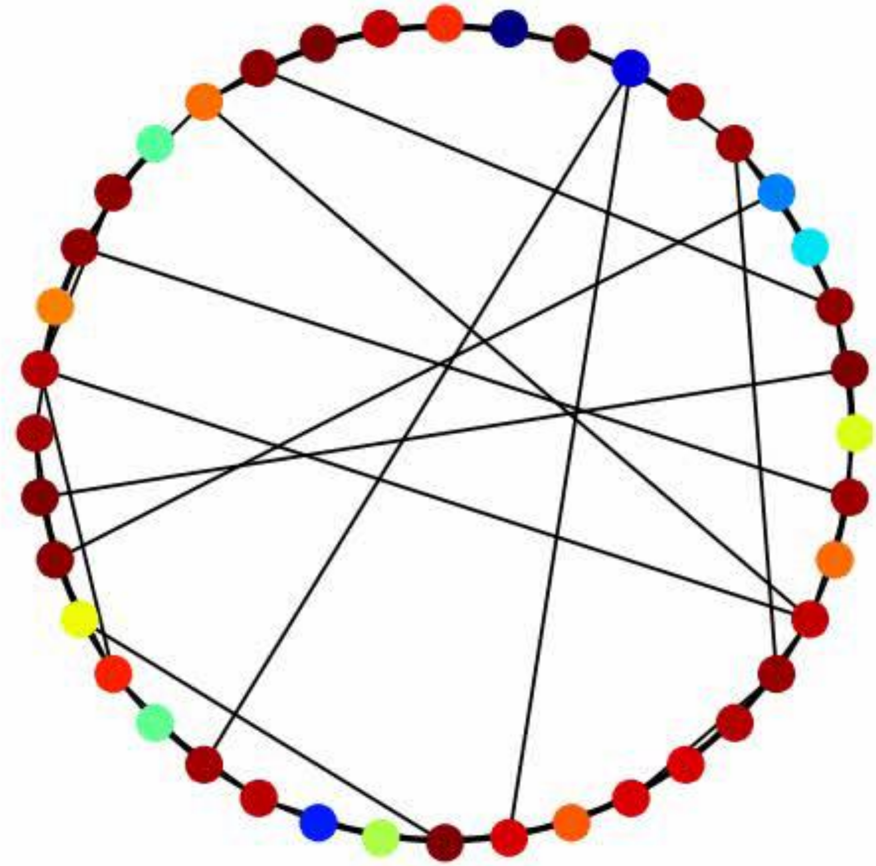


Coupled Logistic Maps

Kaneko, (1989)

$$x_{n+1}^{i} = (1-\varepsilon)\ f\left(x_n{}^{i}\right) + \frac{\varepsilon}{M_i} \sum_{j}{}_{\, j \in B(i)} f\left(x_n{}^{j}\right)$$

Coupled Logistic Maps



Note: the Network structure is a Small World (Watts & Strogatz, 1997)

Rewiring Algorithm

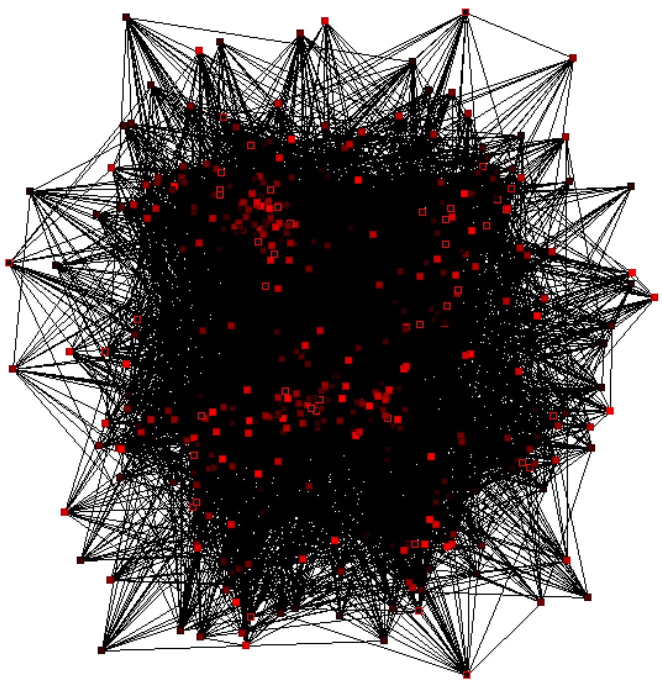
For $\mathcal{N} = \{1, 2, \ldots, n\}$ the set of nodes, $R : \mathcal{N} \times \mathcal{N} \to \mathbb{R}_{0^+}$, ($\mathbb{R}_{0^+}$ the set of non-negative real numbers), we define the following rewiring process:

Step 0: Generate a random graph with n nodes and E edges.
Step 1: Take $x(0) \in [-1, 1]^n$ randomly from a uniform distribution and iterate the dynamics (1) for $t = 0, 1, 2, \ldots, T-1$.
Step 2: Rewiring:

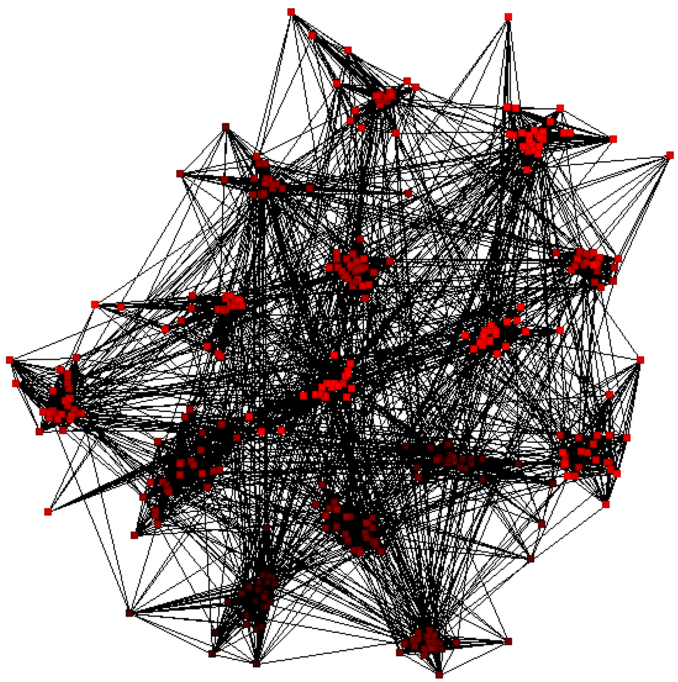
1. select a pivot node $p \in \mathcal{N}$ randomly from a uniform distribution
2. determine the rewiring candidate $c = \arg\min_{j \in \mathcal{N} \setminus \{p\}} R(p, j, T)$
3. go to Step 3 if $c \in \mathcal{N}_p$. If $c \notin \mathcal{N}_p$, update the graph by creating an edge between p and c and removing the edge between p and $\bar{c} = \mathrm{argmax}_{j \in \mathcal{N}_p} R(p, j, T)$.

Step 3: Repeat from Step 1 until 3×10^5 iterates have been reached.

Adaptive Rewiring

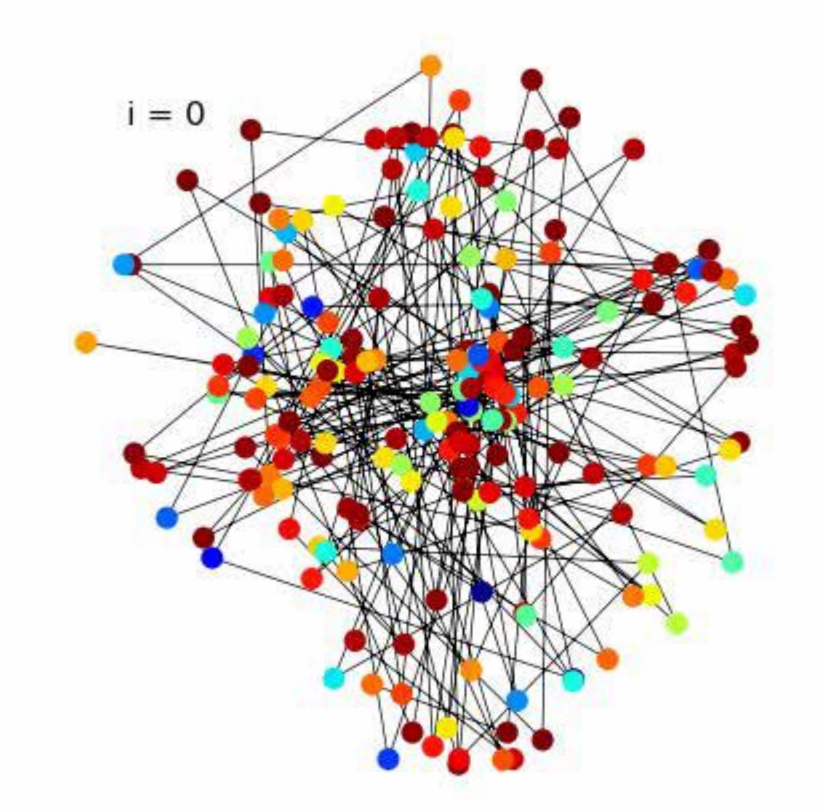


Before



After

Coupled Maps: From Random to Small-world Organization

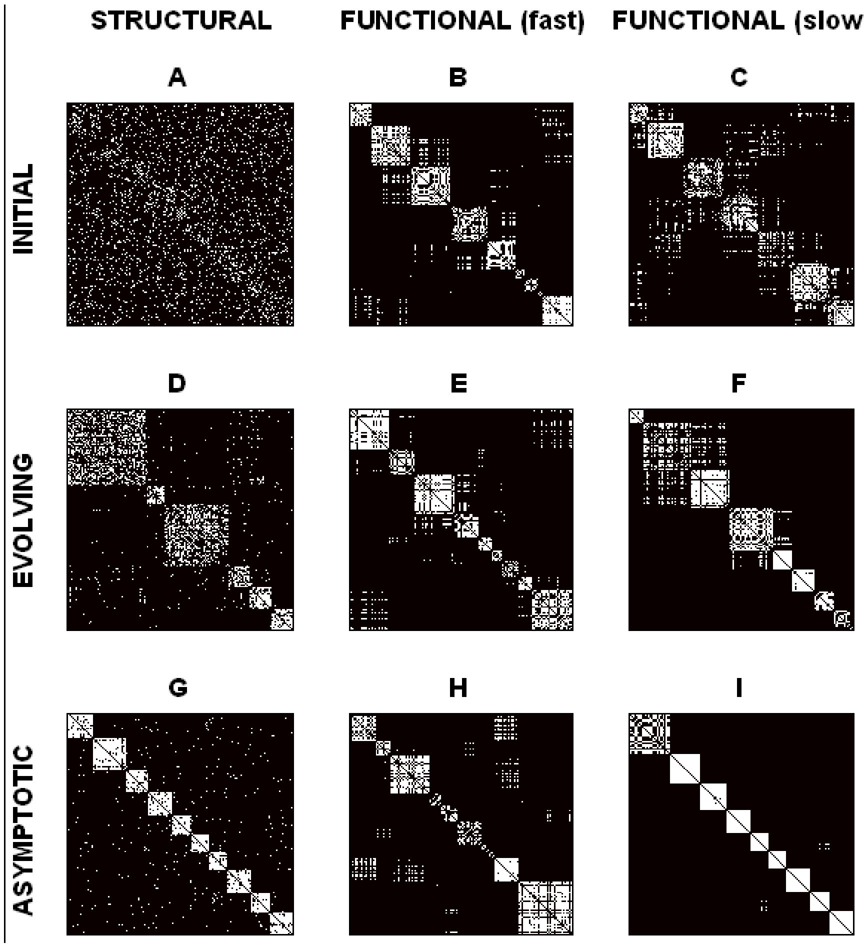


Gong & van Leeuwen, 2003; 2004; Kwok et al, 2007; Rubinov et al, 2009; van den Berg & van Leeuwen, 2004; van den Berg et al., 2012

Network Evolution

Initial (row 1), evolving (row 2) and asymptotic (row 3) network configurations for structural (column 1), fast (column 2) and slow time scale functional (column 3) networks. Fast time scale networks represent the instantaneous patterns of dynamical synchrony. Slow time scale networks based on the correlation coefficient of 100 consecutive functional states. Nodes in all networks were reordered to maximize the appearance of modules,

Rubinov, et al. (2009).



Evidence Suggesting it might work this way

- SWS emerges following spontaneous large-scale wave activity (GDP in prenatal rats) and in cell cultures

- SWS re-emerges in functional architecture following non-REM sleep

Increased Realism

- Model Neurons instead of Coupled Maps
- Network Growth
- Network Pruning
- Spatial Embedding
- Transmission Delays
- …

Increased Realism

- Model Neurons instead of Coupled Maps
- ..
- ..
- ..
- ..
- ...

Kwok et al., 2007

Hindmarsch-Rose Model Neurons

$$\dot{x}_i = y_i - ax_i^3 + bx_i^2 - z_i + I_i + \sum_{j \in A} S_j - V \tag{1}$$

$$\dot{y}_i = c - dx_i^2 - y_i \tag{2}$$

$$\dot{z}_i = r[s(x_i - x_{i_0}) - z_i] \tag{3}$$

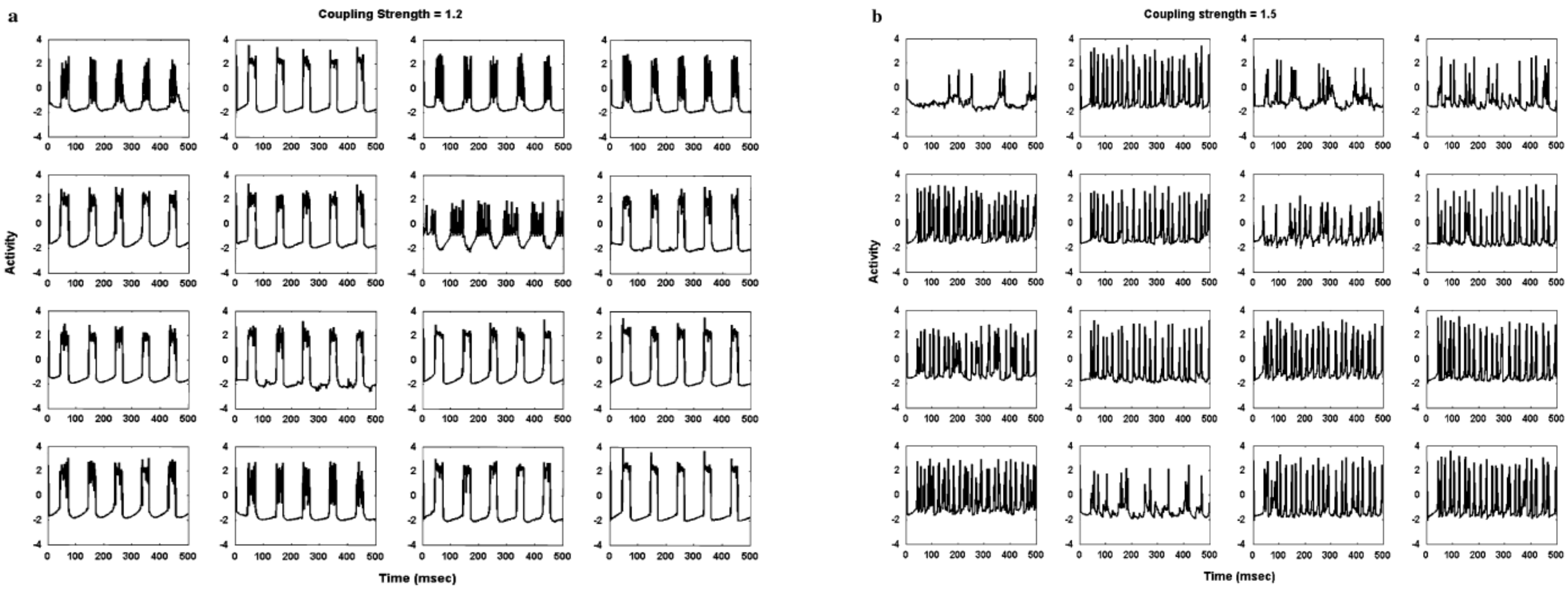
For neurons (x_i, y_i, z_i), with constant *a, b, c, d, r, s*. *A* is the set of neighbors of *i*. S_j and *V* are given by

$$S_j = \Theta(x_j - x^*) \tag{4}$$

$$V = \beta \sum_{j \in \{A,i\}} S_j / N_{A_j} \tag{5}$$

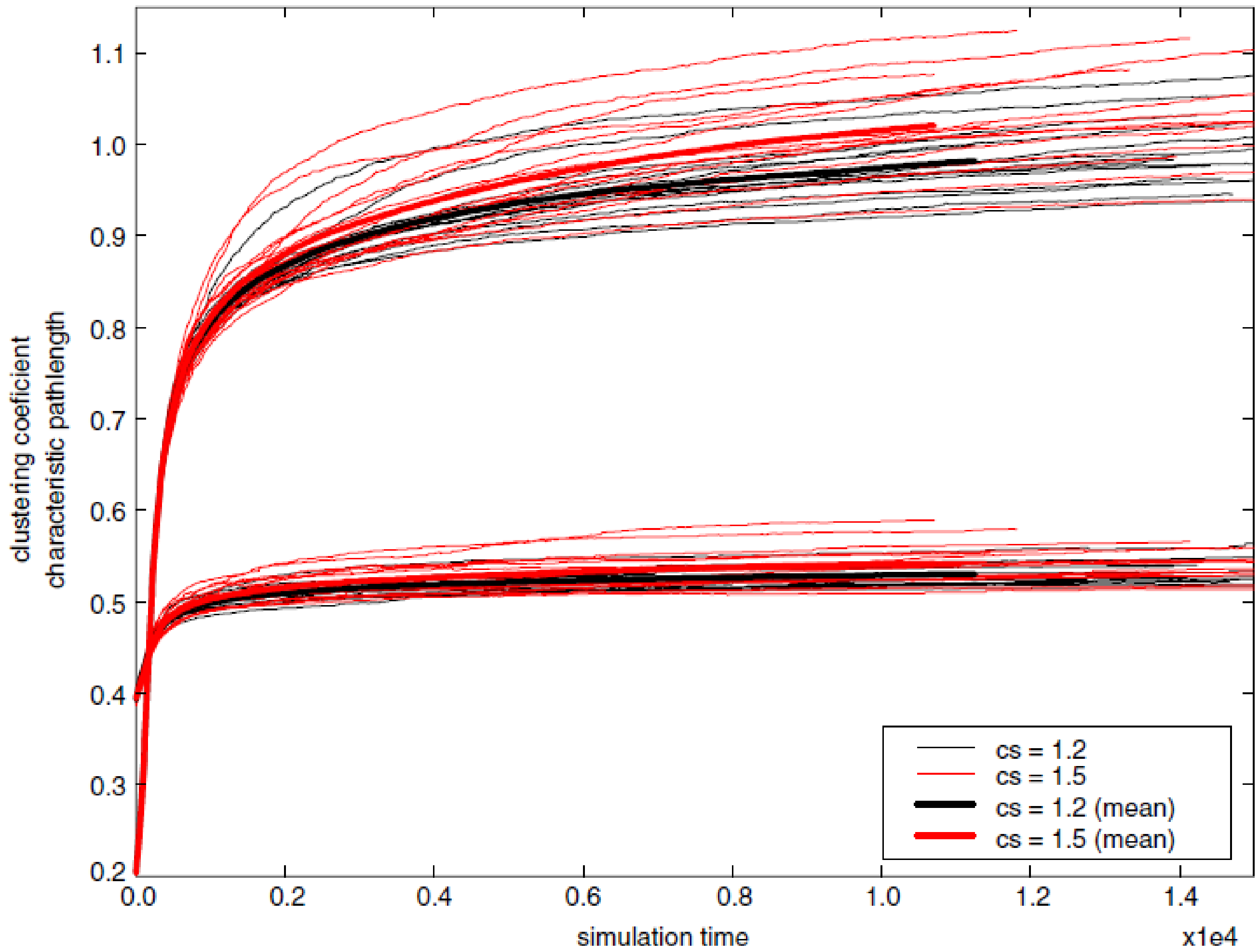
Where Θ is a step function, x* is threshold potential (=0), *β* is coupling strength, and *N* are constants

Activity in a Random Network as a Function of Coupling Strength



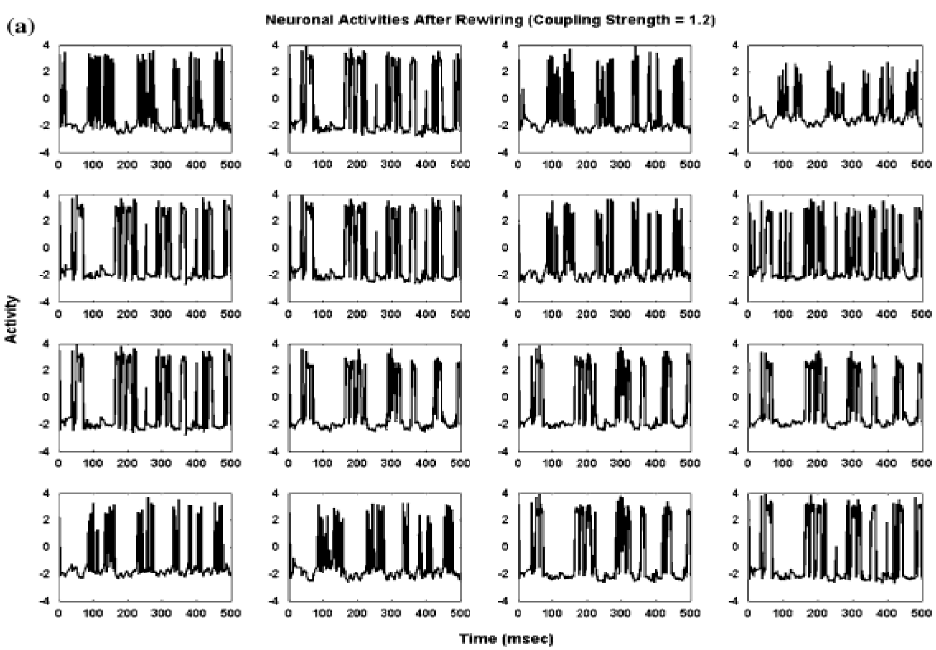
Either periodic bursting or irregular firing, strongly dependent on coupling strength

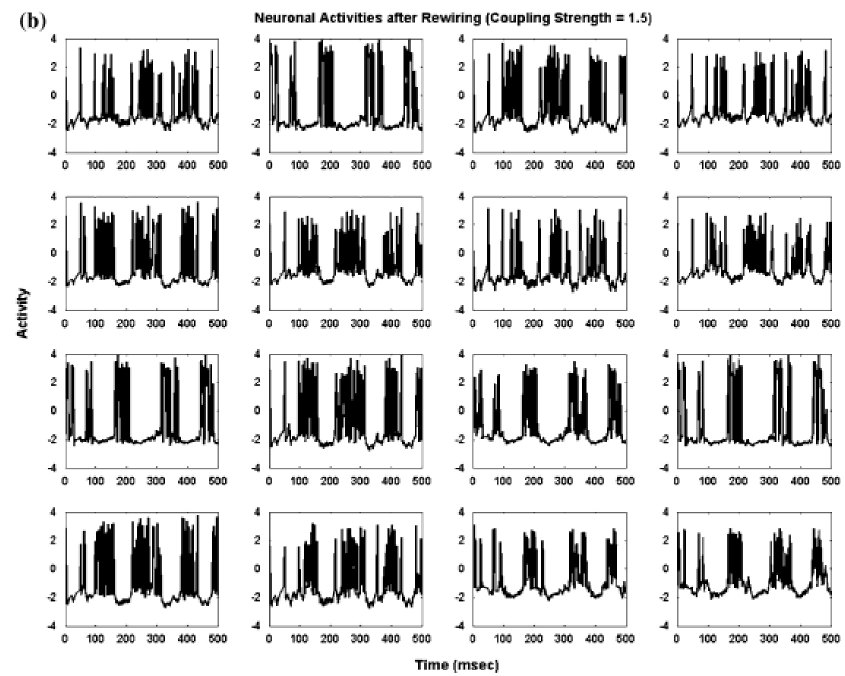
Network Structure Evolution



Note: CC and CPL were normalized against those of a ring

Activity in the Rewired Network as a Function of Coupling Strength





Typical alternation of active and quiet periods, more characteristic of the brain, independent of coupling strength.

Adaptive Rewiring With Model Neurons

Influence of the network structure on the patterns of activity

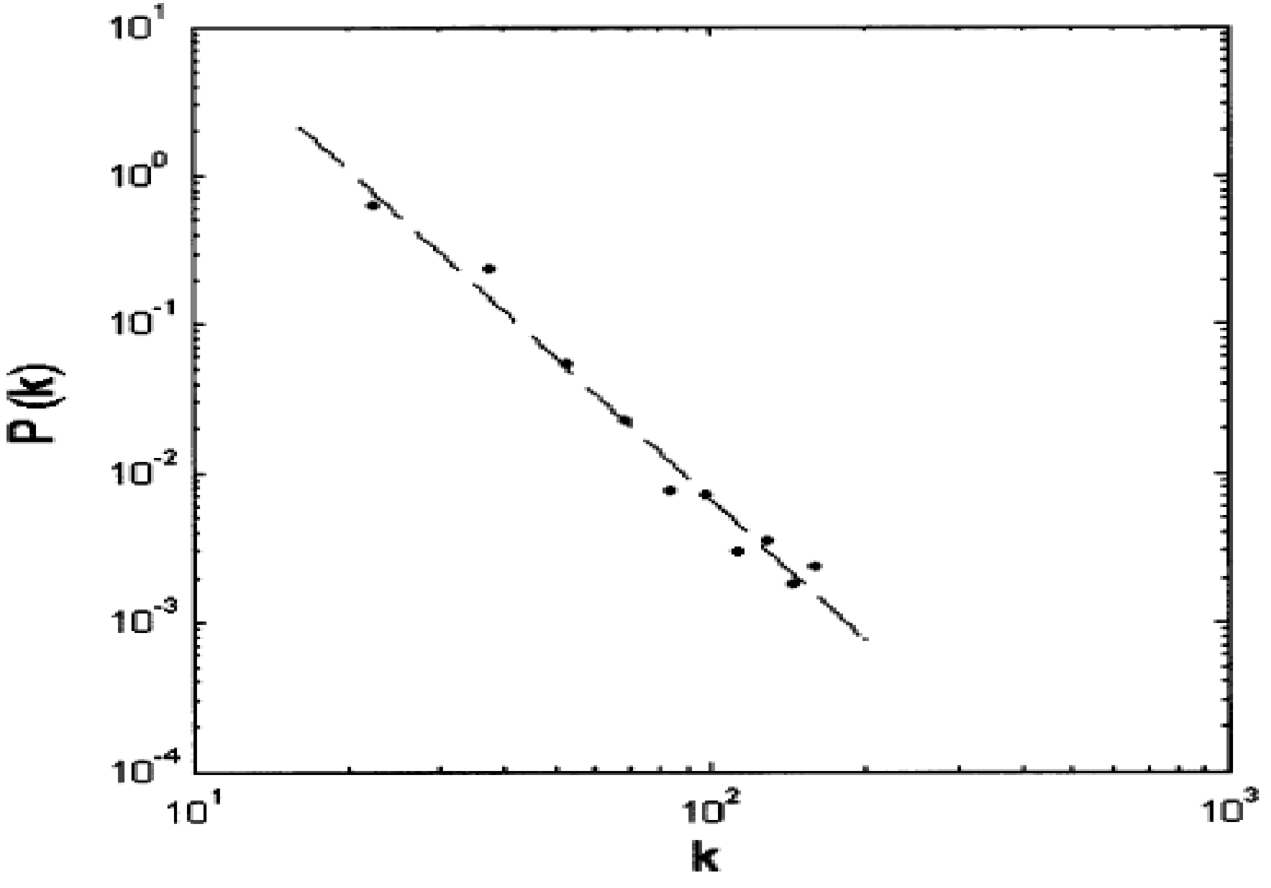
- Initial states are impoverished, and strongly dependent on coupling strength
- Final states after rewiring are interesting, and robust against variation in coupling strength

Increased Realism

- ..
- Network Growth
- ..
- ..
- ..

Gong & van Leeuwen, 2003

Distribution of Connections



Result of rewiring for a network of initially 50 nodes and 850 connections, random insertions at every 75 th iteration.

Adaptive Rewiring With Neuron Growth

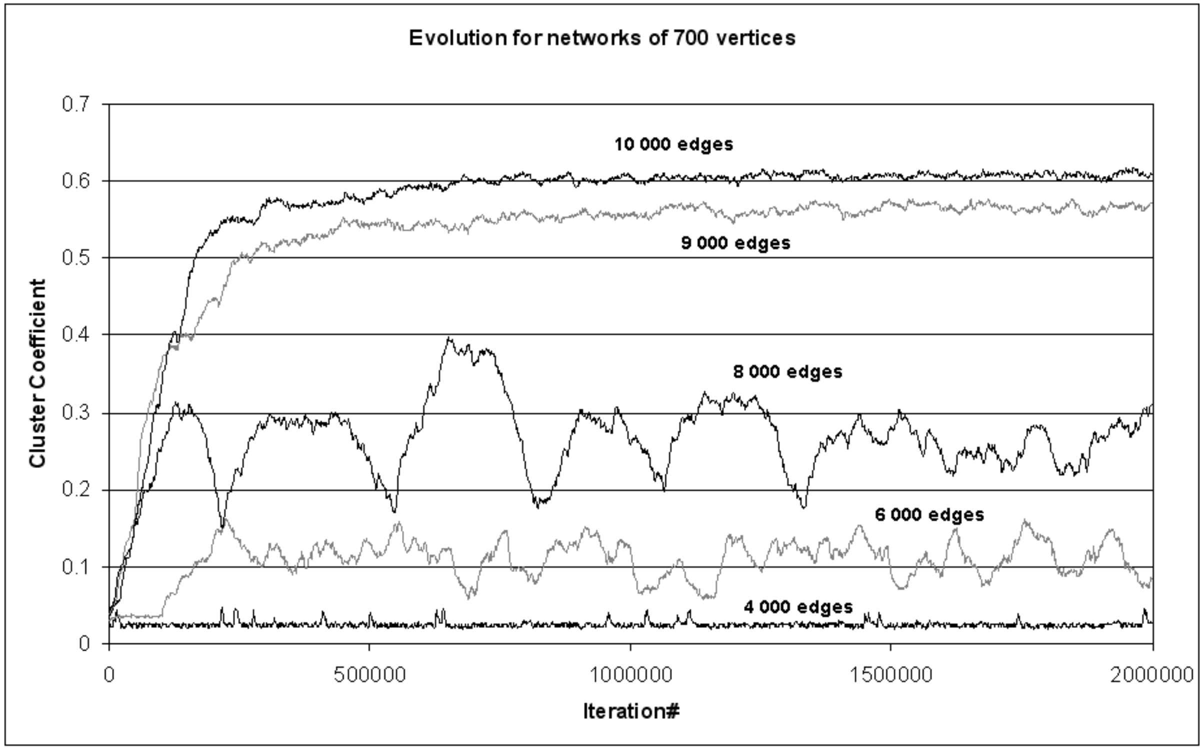
Provides a small-world network with a scale-free connectivity distribution

Robustness of scale-free networks against random lesioning is well-known

Increased Realism

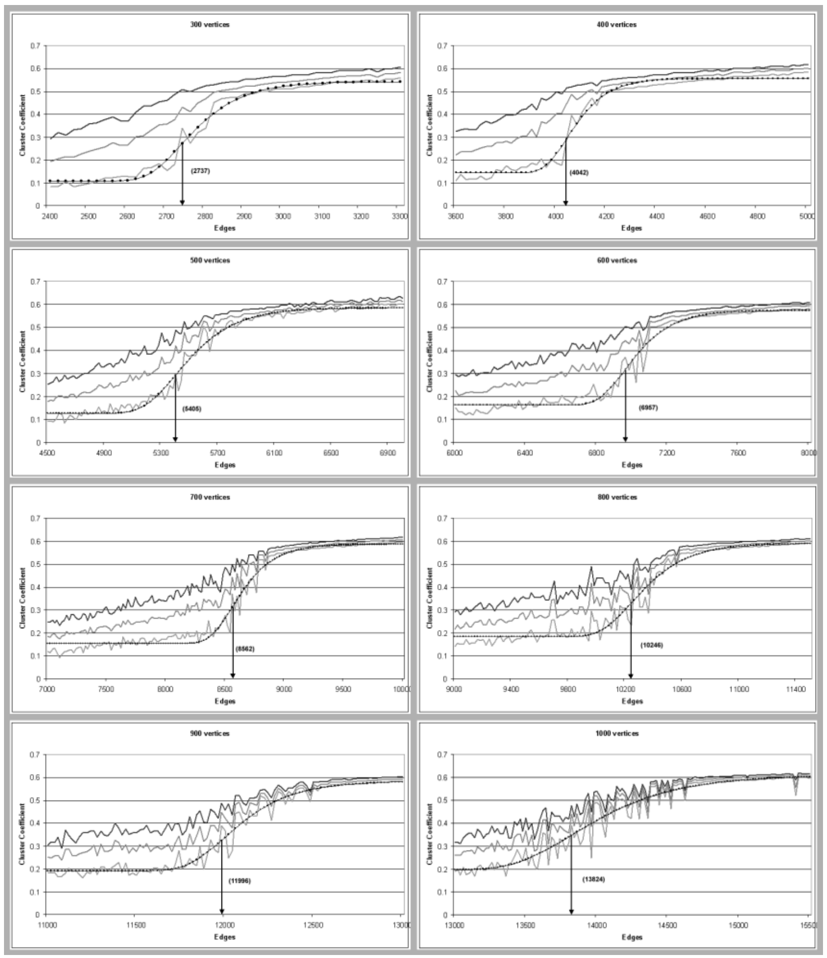
- ..
- ..
- Network Pruning
- ..
- ..
- …

Connectivity is Critical



Self-organization from random to small-world critically in a network of 700 vertices. The self organization occurs through adaptive rewiring. Whether a small-world emerges depends on the number of edges.

Scaling with percolation threshold



Universal scaling in the clustering threshold for self-organized small-world networks.

connectivity needed for SW properties to emerge scales with a universal power α=1.17 to the percolation function in random networks

Grey lines represent minimal, maximal and average values for clustering, the dotted line is the function fitted to the minimum; the arrow indicates its anchor point with the corresponding number of edges in parentheses.

Adaptive Rewiring with Pruning

Critical Threshold Connectivity for Healthy organization into a Modular Small World

Breakdown of local modularity before a breakdown of the global percolation

Increased Realism

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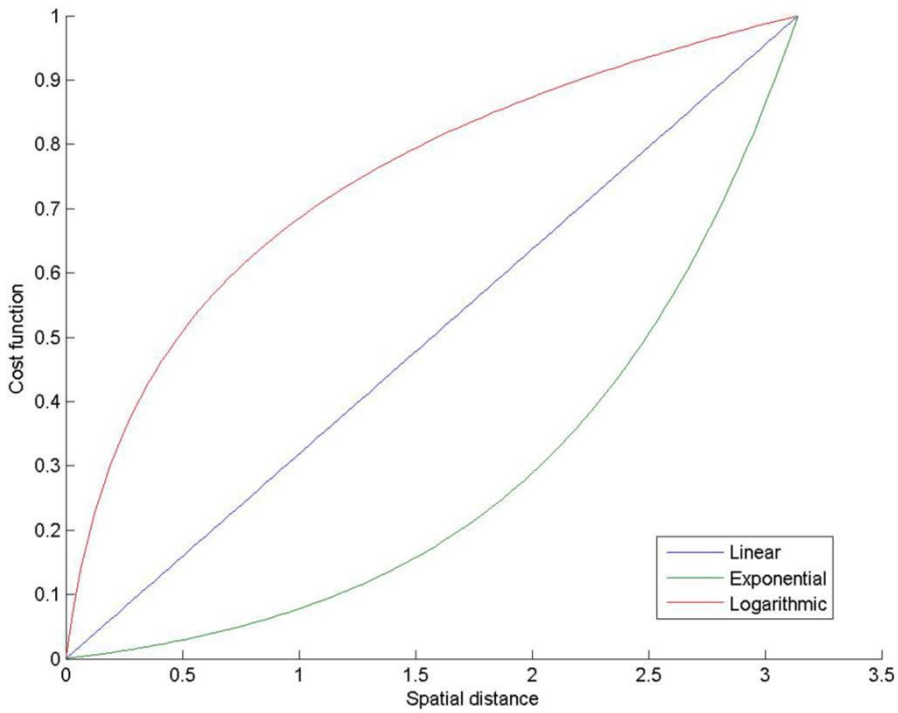
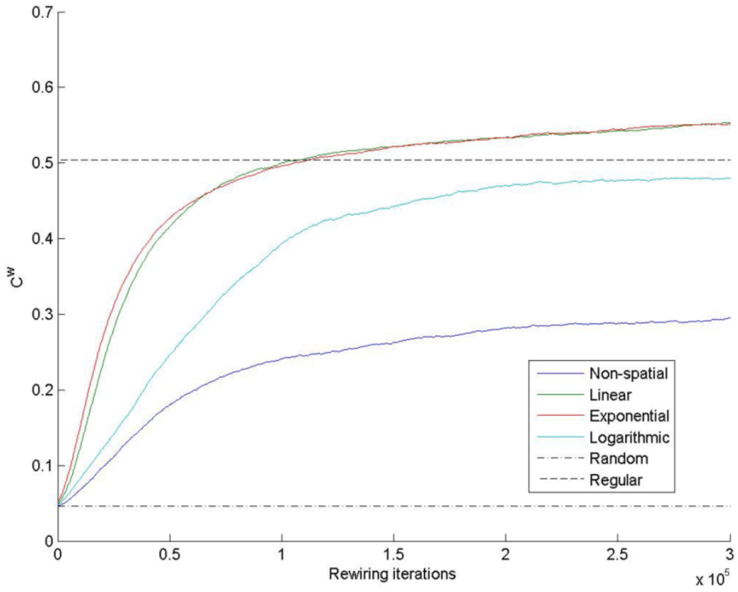
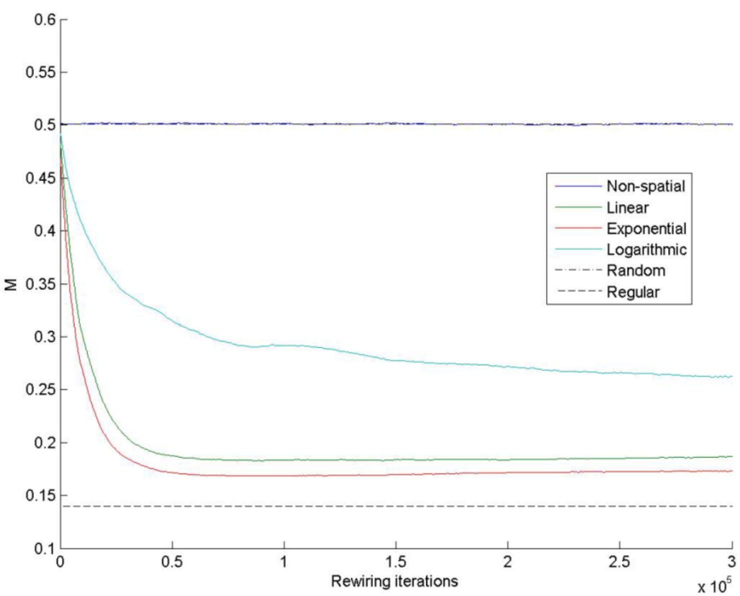
- ..
- ..
- Spatial Embedding
- ..
- ...



Cost functions of spatial distance: linear in blue, exponential in green, logarithmic in red.

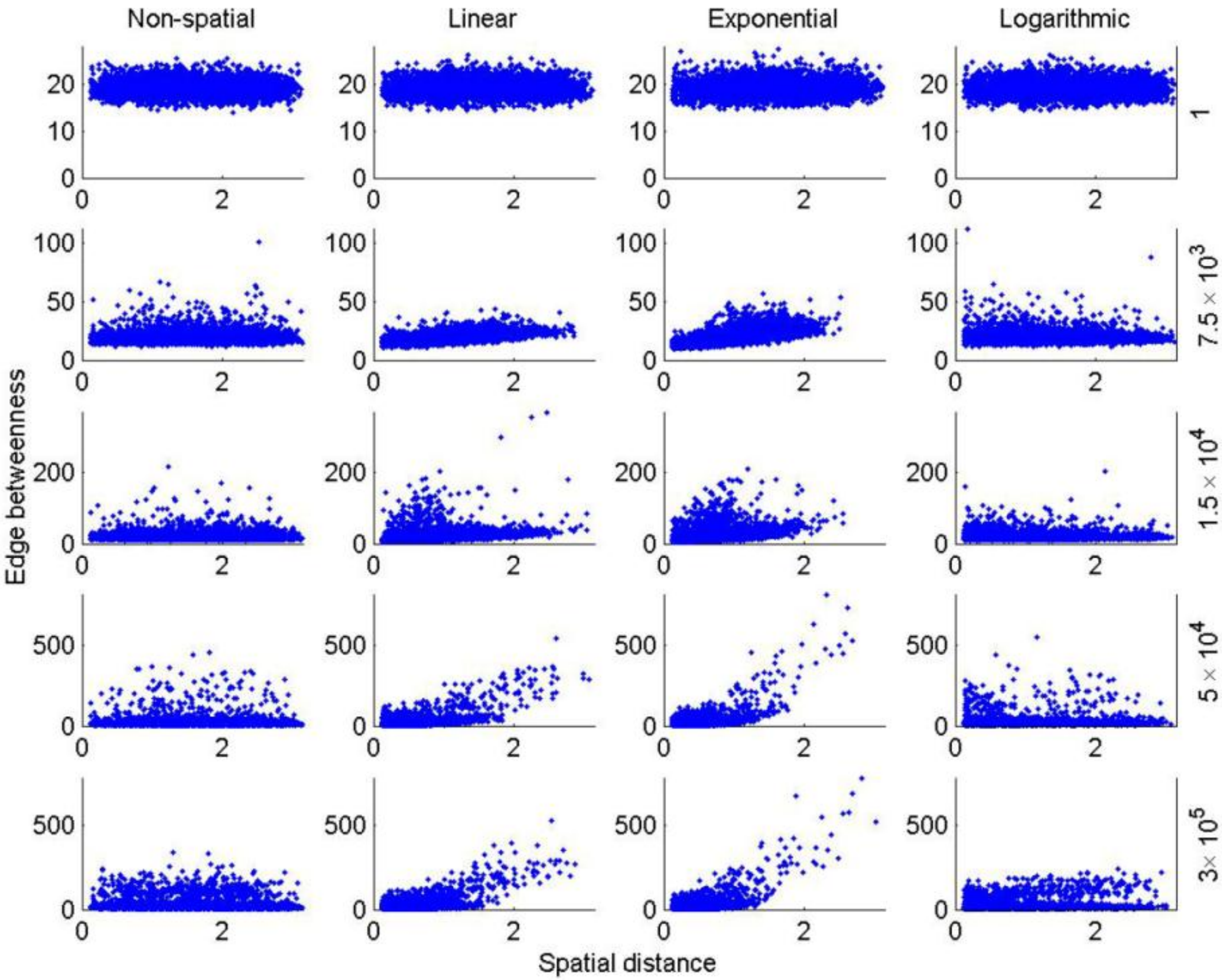


A

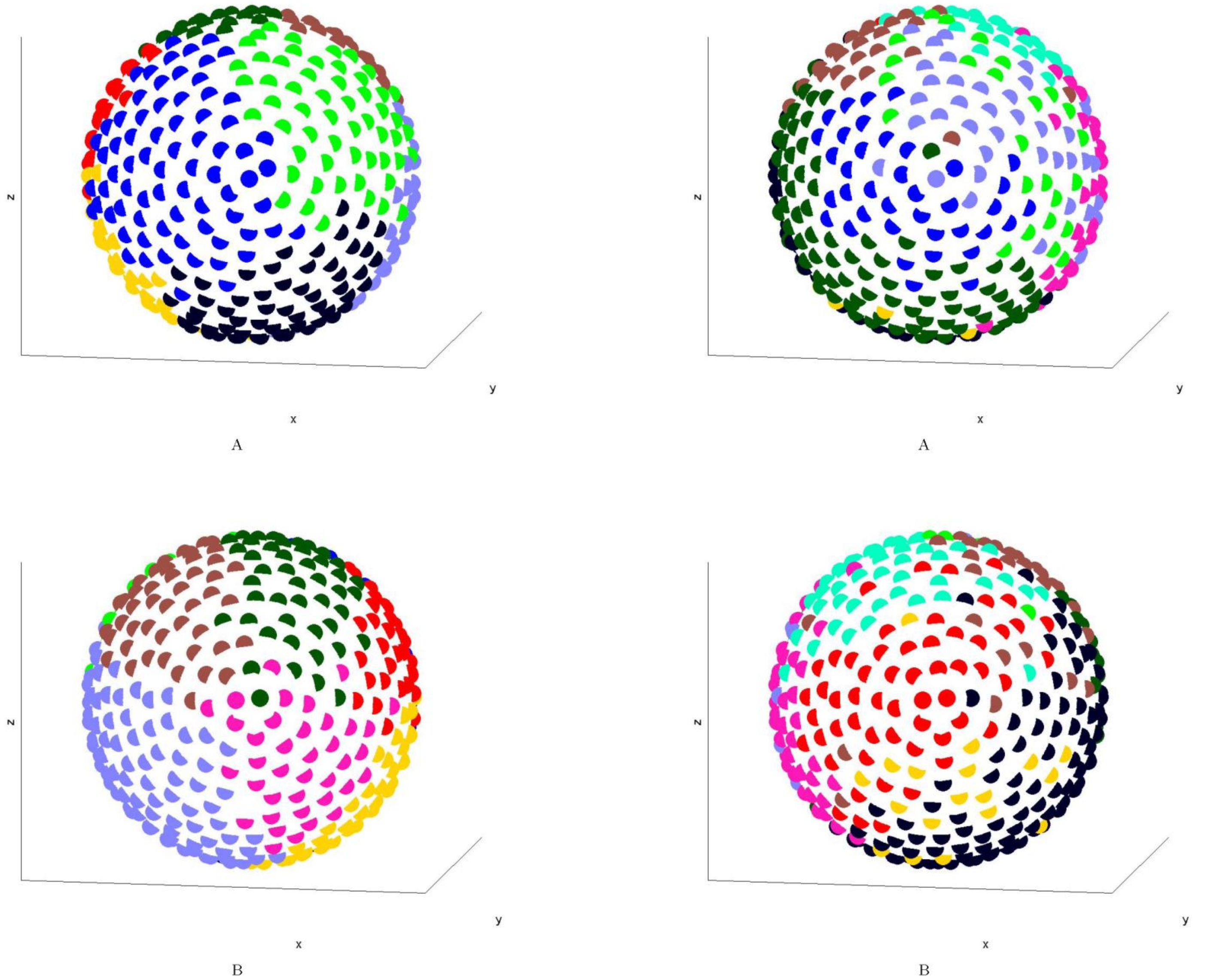


B

Evolution of a the spatially-weighted clustering coefficient values Cw averaged over five runs; and b the network wiring cost values M averaged over five runs; for the non-spatial and spatial rewiring processes, regular lattice on the sphere, and random network



Scatter plots of edge betweenness versus spatial distance. Betweenness values presented here were obtained by uniformly randomly selecting 4 % of nodes from the combined five runs and plotting the betweenness values of all their connections. Rows top to bottom for rewiring steps, 1; 0:75 x 10**3; 1:5 x 10**4; 5 x 10**4; 3 x 10**5, columns are for different rewiring processes



Linear

Logarithmic

Community structure after one run of rewiring. a, b Opposite hemispheres. Coloring indicates modules

Conclusions

Modular Small Worlds

- The brain shows patterned activity
- The brain is a modular small world
- These two are interrelated: a structure emerges adaptively that makes the activity patterns more robust
- Brain diseases linked to disturbance of modular small-world functional architecture: Schizophrenia, Alzheimer, Autism(?)

Symbiosis of Structure and Function: a theoretical model

- Wave sequences help shape the architecture
- The architecture sustains wave sequences

Thank You!

Daan van den Berg, Michael Breakspear, Pulin Gong, Nicholas Jarman, Peter Jurica, Hoi Fei Kwok, Michael Rubinov, Chris Trengove, Ivan Tyukin, Erik Steur

www.perceptualdynamics.be