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LETTER TO THE EDITOR

NONARBITRARY REGULARIZATION OF ACOUSTIC SPECTRA

Alexander N. Gorban
Iliya V. Karlin
Computing Center,
Krasnoyarsk, 660036 Russia

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We introduce a method for improving the Chapman-Enskog expansion (CE) free of recipe assumptions. Obstacles of the CE method are well known, for example, a short-wave instability of the Burnett approximation [1]. Many attempts were made to improve the CE expansion. In particular, we used the idea of partial summing [2, 3]. However, all these attempts have an ad hoc character. Hence, it is important to develop a more fundamental method of improving the CE expansion.

The famous KAM theory [4-6] might serve for a pattern. One can consider it as an improved perturbation theory. There one applies rapidly converging Newton method instead of Taylor expansion, and one searches for a dynamic invariant manifold rather than for a solution.

Here we suggest a method of constructing dynamic invariant manifolds for the Boltzmann (B) equation. Same as in KAM, we will use the Newton method. Each iteration will be concordant with the H-theorem.

Our method consists of two main parts:

1. A constructing of a specific thermodynamic parameterization for an arbitrary manifold which gives dynamic equations on this manifold (this part lacks in the KAM

theory, and it is caused by the request on concordance of every approximation with the H-theorem).

2. A correction of the dynamic noninvariance of a manifold by the Newton method.

We will describe the method for a general dynamic system with a global convex H-function. The manifold $\{f(a)\}$ consists of distributions $f(a)$, where a are coordinates on the manifold. The request 1 means that one should define operators $M(f)$ for $\{f(a)\}$ so that:

$$\mathrm{d}H(M(a))/\mathrm{d}t=(\nabla_{M(a)}H(M(a)),\ \mathrm{d}M(f(M(a)))/\mathrm{d}t)\leqslant 0;$$

$$H(M(a))=H[f(M(a))],\quad \mathrm{d}M(f(M(a)))/\mathrm{d}t=\nabla_f M\cdot J(f)\Big|_{f=f(a)} \qquad (1)$$

Here $J(f)$ is a dissipative vector field (the collision integral for the B equation), $p\cdot q$ is a scalar product (with integration over velocities for the B equation), $(.,.)$ is a scalar product for macroscopic parameters.

The only suitable parameterization is obtained via the functionals (the only restriction: $\{f(a)\}$ is not tangent to a level of the H-function for any $f(a)$):

$$M^*_{f(a)}(f)=\mu(f(a))\cdot f,\qquad \mu(f(a))=\nabla_f H[f]\Big|_{f=f(a)} \qquad (2)$$

Here $\mu(f)=\ln f$ for the B equation. Thermodynamic parameterization for $\{f(a)\}$ is obtained by functionals $M^*_{f(a)}(f)$ and by adding any other functionals suitable for the dimension of $\{f(a)\}$. If the manifold consists of quasi-equilibrium distributions then one does not need a search for new functionals $M^*_{f(a)}(f)$.

The condition of dynamic invariance is easily formalized for the manifold $\{f(M(a))\}$:

$$\Delta(f(M))=(\nabla_M f(M),\ \partial M(f(M))/\partial t)-\partial f/\partial t\Big|_{f=f(M)}=0 \qquad (3)$$

Here $\partial f/\partial t$ is $-\vec{v}(\partial f/\partial\vec{x})+J(f)$ for the B equation, and $\partial M(f(M))/\partial t=(\nabla_f M\cdot\partial f/\partial t)\Big|_{f=f(M)}$. One can solve (3) by the Newton iterative procedure.

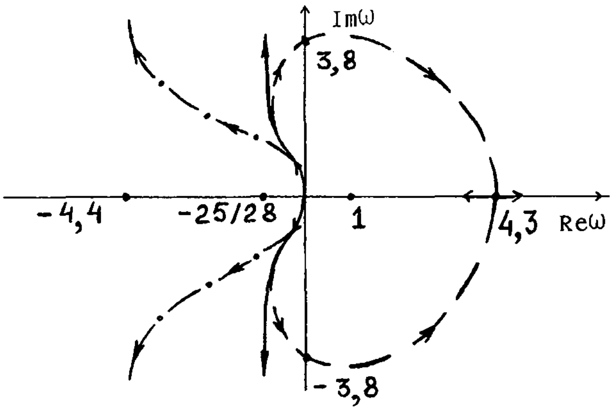

FIG. 1. Dispersion curves for (5) (—), for the Burnett appr. [1] (– –), and for the partial summing [2] (–·–). Arrows indicate an increase of k^2.

The method is an iterative algorithm: 1) choose an initial manifold and define dynamics on it using the functionals (2); 2) linearize (3) near the initial manifold and obtain the first correction; 3) define dynamics on the manifold obtained using corresponding new functionals (2), etc.

Let the initial manifold consists of local Maxwell distributions f_0. Equation for the first correction is:

$$L[f_0,\delta f_1]+K[f_0,\delta f_1]=\Delta(f_0)\equiv f_0\Big\{(2k_B/Tm)^{1/2}\vec{c}\,(c^2-\tfrac{5}{2})\frac{\partial T}{\partial\vec{x}} + 2(\vec{c}\vec{c}-\tfrac{1}{3}\delta_{ij}c^2)\frac{\partial\vec{u}}{\partial\vec{x}}\Big\},\quad K[f_0,\delta f_1]=\sum_{i=0}^{i=4}\frac{\partial f_0}{\partial M_i}\int\psi_i\vec{v}\frac{\partial\delta f_1}{\partial\vec{x}}d^3v-\vec{v}\frac{\partial\delta f_1}{\partial\vec{x}} \quad (4)$$

Here $L[f_0,\delta f_1]$ is the linearized collision integral, ψ_i are collision invariants. For a small deviation from equilibrium, equation (4) yields the following tension tensor σ and the heat flux q (one-dimensional case, T and u are dimensionless deviations [2]):

$$\sigma=-\tfrac{2}{3}n_0T_0R\{2(\partial u/\partial x)-3(\partial^2T/\partial x^2)\};\quad R=(1-(2/5)\partial^2/\partial x^2)^{-1}$$

$$q=-(5/4)T_0^{3/2}n_0R\ \{3(\partial T/\partial x)-(8/5)(\partial^2u/\partial x^2)\} \quad (5)$$

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