Non-uniform Small-gain Theorems for Systems with Unstable Invariant Sets and Their Applications

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We consider the problem of asymptotic convergence to invariant sets in interconnected nonlinear dynamic systems. Standard approaches often require that the invariant sets be uniformly attracting, e.g. stable in the Lyapunov sense. This, however, is neither a necessary requirement, nor is it always useful. Systems may, for instance, be inherently unstable (e.g. intermittent, itinerant, meta-stable) or the problem statement may include requirements that cannot be satisfied with stable solutions. This is often the case in general optimization problems and in nonlinear parameter identification or adaptation. Conventional techniques for these cases rely either on detailed knowledge of the system's vector-fields or require boundeness of its states. The presently proposed method relies only on estimates of the input-output maps and steady-state characteristics. The method requires the possibility of representing the system as an interconnection of a stable, contracting, and an unstable, exploratory part. We illustrate with examples how the method can be applied to problems of analyzing the asymptotic behavior of locally unstable systems as well as to problems of parameter identification and adaptation in the presence of nonlinear parametrizations. The relation of our results to conventional small-gain theorems is discussed.