MA1221

PURE MATHEMATICS AT WORK 2006/07

WORKSHOP: GRAPH THEORY

Work suggestions

In the following I give you a number of a suggestions about what you should think in relation to the problems Q1 - Q3. Start with S1. Note that when you get stuck you can sometimes get hints by reading the next suggestions. Also, some of them are independent of previous ones. So after having done the first few suggestions to get into the topic we want to study, you can jump some numbers and try suggestion further below.

S1. We next want to see how to attack these problems in a systematic way which allows us, for example, to solve generalizations of these problems as well. The concept needed here is that of a graph. Relate each problem with a graph. Can you reformulate the problems above (using graphs) in a more abstract language? Again, it is not necessary to work out all details.

S2. Draw different graphs. Can you draw graphs with different properties? Compare with the graphs in S2. Discuss your drawings and then give a formal (mathematical) definition what a graph is. As a hint: most of the time mathematicians like to think in terms sets.

S3. *Isomorphisms of graphs* Discuss whether the following graphs are all the same. If they are the same try to formalize (i.e. using mathematical language) when you consider two graphs to be the same.

Draw more examples of graphs which you consider to be the same, but also graphs which you consider to be different. When trying to formalize when two graphs are the same, you will need to use the answer of S2, and you might want to modify your answer after having thought about this problem. Compare this question with S4 and S5.

S4. Draw different graphs with three vertices. Draw all graphs with four vertices. Hint: Your answer depends on which definition of a graph you want to use. If necessary, change it to one which gives you a finite list in each case.

S5. *Planar graphs* The following are called K_3, K_4, and K_5, the complete graphs of order three, four and five, and are given by the following pictures

Obviously, K_3 is drawn without any intersecting lines. Can you draw K_4 and K_5 without any intersections? Note that this question is related to S3, and you might want to change your definition of when you consider two graphs to be the same. Have you already formulated Q2 in more mathematical language?

S6. Can you draw K_6? Define what you understand by the complete graph K_n of order n. Find a formula for the number of edges of K_n. Prove your formula by induction.

S7. *Euler walks and Euler circuits.* Consider the graphs associated to the different supplied plans of cities. Can you draw them without lifting your pencil and without retracing any edges? Is it easier if you are not required to end at the same point where you started? Try to find invariants which explain your answers.

S8. Take a graph. We define the degree of a vertex to be the number of edges leaving from this vertex. Determine the degree of each vertex in the graphs you have drawn in S8. Give a conjecture for when a graph can be drawn without lifting the pencil and retracing any edges, and when not.

Once you have worked to the end of this sheet you possibly will neither have solved the problems Q1 - Q3 nor have solved every question the suggestion sheet, but you should have some idea what graphs are and how problems can sometines reformulated in terms of graph theory. You will get more answers in the lectures. If you are impatient to know more you may want to look in any book on discrete mathematics for a chapter on graph theory. You can also look into books on graph theory which go much deeper than the outline covered in this handout.