MA1221

PURE MATHEMATICS AT WORK 2006/07

Question Sheet I

The work from this question sheet counts 15% towards the final mark of the is module and will be collected at the 14:30 lecture/workshop on Monday, 4 December. Late work will not count.

1. Let F_n denote the n-th Fibonacci number. Prove by induction that

$$F_1^2 + F_2^2 + ... + F_n^2 = F_n F_{n+1}.$$

2. The Lucas sequence is closely related to the Fibonacci sequence. It is defined by $L_{n+1} = L_n + L_{n-1}$ for $n \geq 2$ and $L_0 = 2$ and $L_1 = 1$. By solving the recurrence relation, find a closed formula for L_n.

3. (Propagation of annual plants) Annual plants produce seeds at the end of their season (September) and then die. Seeds lie dormant in the ground until next spring (for 6 month) Then some of them germinate and create new plants, others lie in the ground for an additional year (18 month altogether) and then germinate. Seeds which which have not germinated after 18 month will die. A plant produces lots of seeds and experiments showed that in the average 9/5 seeds per plant germinate after 6 months and 2/5 seeds per plant after 18 months. Let p_n denote the number of plants at the end of the n-th season. Let $p_0 = 100$ and $p_1 = 182$. Find a recurrence relation for p_n as well as a closed formula formula.

4. (Recurrence relation of first order) Consider the sequence defined by

$$u_k = au_{k-1} + b$$

with initial term u_1. Prove by induction that

$$u_k = a^{k-1}u_1 + b(1 + a + a^2 + ... + a^{k-2})$$

for $k \geq 2$. Thus, when $a \neq 1$ we have

$$u_k = a^{k-1}u_1 + b(\frac{a^{k-1} - 1}{a - 1})$$

Calculate u_{10} for the sequence defined by $u_k = 2u_{k-1} + 3$ with $u_1 = 5$

5. Up to isomorphism, i.e. up to same form, draw all simple graphs with 6 vertices and 3 edges. Give an argument why your graphs are not isomorphic and why you got all.