

MA1221

PURE MATHEMATICS AT WORK 2006/07

WORKSHOP: GRAPH THEORY

Instructions: You will work in groups of at most four people. While you are working on the problems and the suggestions I will go around and be available for discussion. By the end of the end of the work shop session every group should hand in answers for some of the questions S1 - S9 and, if you are able, to the problems Q1 - Q3. Please note that it is very unlikely and it would be much to much to ask for that you will solve everything. Pick those things which you find attractive.

This work will be assessed and contributes 15% to the final mark.

Problems

Q1. *The odd party problem.* An old fellow wishes to have an party that is attended by an odd number of people, each of whom is acquainted with an odd number of other people at the party. Can this odd situation occur. Note that if A knows B , then we assume that B knows A . Hint: It might help to specialize the problem: is there an odd party for 3 people? for 4? For 5? If not, why not?

Q2. *The three utilities problem.* Suppose we have 3 utility poles (say phone, electricity and water) and 3 houses. Suppose each household wishes to be connected to each of the utilities. Can this be done if wires cannot cross each other?

Q3 *The Königsberg bridge problem.* Consider some of the plans of fictional cities supplied and try to trace a walk starting where the person in the diagram is standing, passing each bridge once and returning to the starting point. Such a walk is called an *Eulerian circuit*. Can you do that for the different plans?

(Sheet A represents the city of Königsberg, namely the bridges over the river Pegel which runs through the city and has two islands in the middle.)

Does it make difference if we do not insist on coming back to the starting point. Can you then find a route crossing every bridge exactly once? This is called an *Eulerian walk* rather than a circuit. Does it make a difference where yo start?

Start by trying to solve some of the above questions by your own ad-hoc methods. This will help you to familiarize with the problems. Only spend a few minutes on this. I will then supply a sheet with some work suggestions how to attack these questions in a systematic way.